

Session - 2024 - 25

Sub: Strength of Material (3rd sem)

Prepared by:-

Pratik Kr. Gupta.

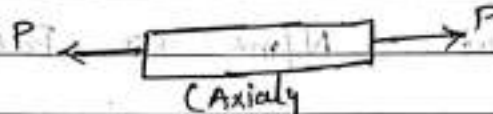
Unit-I

Direct stresses and strain in components.

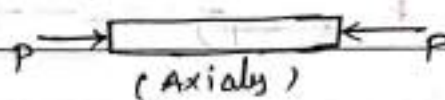
Load:- A load may be defined as a force tending to effect, produce deformation, stresses or displacement in the structure.

Load may be classified by number of way


(A) (i) ~~tensile load~~ (ii)(A) ~~in dead load~~ ~~or fluctuating load~~


(A) (i) tensile load:-  $P = \text{tensile load}$

(ii) compressive load:-

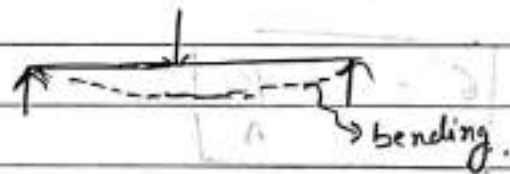
 $P = \text{compressive load.}$

(iii) shearing load:-

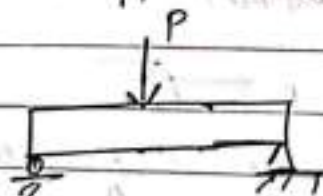
 $P = \text{shearing load}$

(iv) Torsional load (twisting load):- 

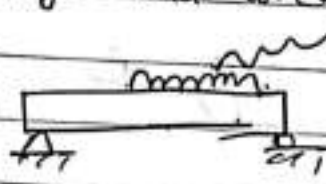
(v) Bending load:-



B. (i) Point load:- load, which is applied in small area or in single point.

 $P = \text{point load}$
(N) \rightarrow Unit
Newton.

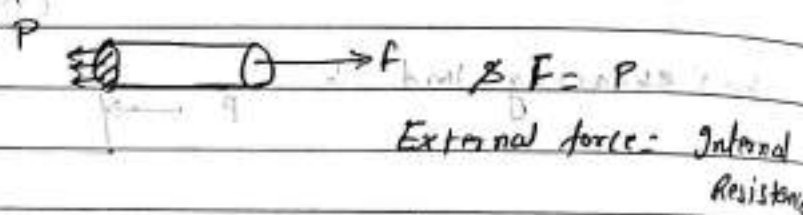
(ii) Distributed load:- load, which is applied over a considerable length and area.

 $w N/m$
 $w = \text{uniformly distributed load.}$
N/m \rightarrow Unit

Stress :- (प्रतिक्रम) :- When a material is subjected to an external force, a resisting force is setup within the material. This internal resistance force per unit area is known as stress.

किसी वस्तु के ईकाई क्षेत्रफल पर कार्य करने वाले आन्तरिक प्रतिरोधी बल को प्रतिबल कहते हैं।

Unit = N/m^2 or Pascal (Pa)

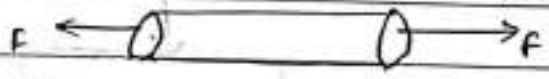


\therefore stress = $\frac{\text{internal resistance}}{\text{Area}}$ = $\frac{\text{आन्तरिक प्रतिरोधी बल}}{\text{क्षेत्रफल}}$

$$\sigma = \frac{P}{A}$$

Types of stresses :-

(i) Tensile stress :- (खींचना)



$$\sigma_T = \frac{F}{A} \text{ N/m}^2$$

σ_T = tensile force stress
 F = tensile force
 A = Area (cross-section Area)

(ii) Compressive :- (दबाना)



$$\sigma_C = \frac{F}{A}$$

σ_C = compressive stress
 F = compressive force

(c) Bending stress:-(d) Shear stress:-

$$\tau = \text{shear stress} = \frac{F}{A}$$

A - Area (Parallel to force)

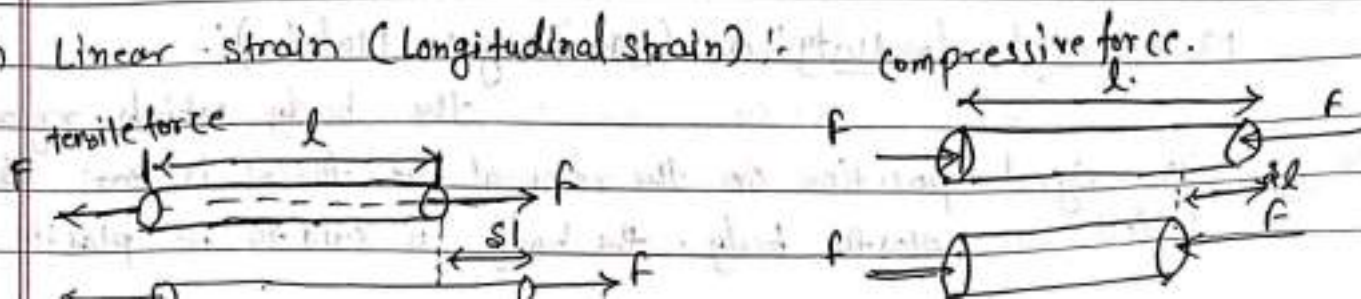
Strain:- (विहति) When a body is subjected to some external force there is some change in dimensions of body. The ratio of change of dimension of the body to its original dimensions is known as strain.

वास्तविकता में प्रभाव में Body में विकृत होता है, या उसके आकार में परिवर्तन होता है। इस परिवर्तन को इस परिवर्तन के माप को ही विहति कहते हैं।

$$\text{Strain } \epsilon = \frac{\text{change in dimensions}}{\text{original dimensions}}$$

Unit = strain की कोई Unit नहीं होती।

Types of strain:-

(i) Linear strain (Longitudinal strain):-

∴ linear strain = $\frac{\text{change in length}}{\text{original length}}$

$$E \text{ or } e = \frac{\delta l}{l}$$

Note:- पहले figure में tensile load लगाने के कारण लम्बाई में वृद्धि हुई तथा दूसरे figure में compressive force लगाने के कारण लम्बाई में कमी आई है।

(ii) Lateral strain:- \odot may be defined as ^{the ratio of} change in diameter to original diameter.

$$E_d \text{ or } e_d = \frac{\text{change in diameter}}{\text{original diameter}} = \frac{\delta d}{d}$$

Note:- पहले figure में tensile force लगाने के कारण व्यास में कमी आई है, तथा दूसरे figure में compressive force लगाने के कारण व्यास (diameter) में वृद्धि हुई है।

(iii) Volumetric strain:- may be defined as the ratio of change in volume to original volume.

$$e_v = \frac{\text{change in volume}}{\text{original volume}} = \frac{\delta V}{V}$$

Modulus of elasticity:- or (~~you~~ Young's Modulus):-

the body which regain its original position on the removal of forces is ~~not~~ called as elastic body. the body is said to be plastic

if the strain exist even after the removal of external forces. there is always a limiting value of load upto which strain totally disappears on the removal of load - the stress corresponding to this load is called elastic limit.

Robert Hooke discovered that within elastic limit stress is directly proportional to strain. i.e.

stress \propto strain

$$\frac{\text{stress}}{\text{strain}} = \text{a constant}$$

this constant termed as

Modulus of elasticity (E).

$$\text{Stress} = E \times \text{Strain}$$

$$\sigma = E \times \epsilon$$

E = modulus of elasticity or Young's modulus.

Unit - N/m^2 or Pa.

Modulus of rigidity (G or C) may be defined as the ratio of shear stress (τ) to shear strain (ϕ). and denoted by (G or C).

$$\text{modulus of rigidity (G or C)} = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

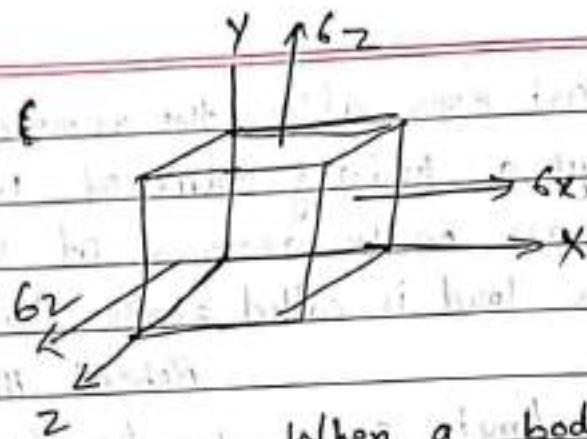
$$G = \frac{\tau}{\phi}$$

* F = shear load.



in case of shearing load, a shear strain will be produced which is measured by the angle through which the body distorted.

Bulk Modulus :- K



When a body is subjected to three mutually perpendicular stress of a equal intensity the ratio of direct stress to the corresponding volumetric strain is known as Bulk modulus.

$$\text{Bulk modulus } (K) = \frac{\sigma}{e_v}$$

Unit - N/m^2

from Hook's law:-

$$\sigma \propto e$$

$$\sigma = E \times e$$

where

$$\sigma = \text{stress} = \frac{P}{A}$$

$$E = \text{modulus of elasticity} \quad e = \text{strain}$$

put the values.

$$\frac{P}{A} = E \times \frac{\Delta l}{l}$$

$$\Delta l = \frac{P \times l}{A \times E}$$

where:-

Δl - change in length

P - load.

A - Area.

E - Modulus of Elasticity.

2nd

Poisson ratio :-

Time Table 4th and 6th Semester

& for 6th SEM - "MF 02"

2 nd	3 rd	4 th	5 th	6 th
11:30-12:30	12:30-01:30	01:30 TO 02:00	02:00-03:00	03:00-04:00
MM	MP		MP	

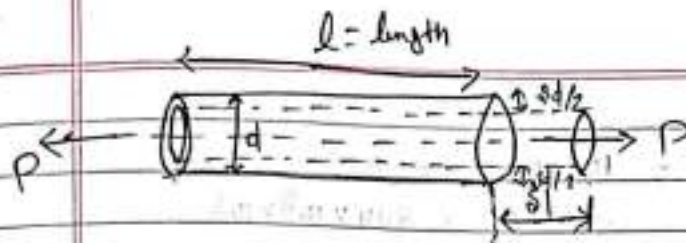
W.E.F 6th SEM Date 06/01/2025
W.E.F 4th SEM Date 03/02/25

Time Table for 3rd SEM - "MF 01" & for 5th SEM - "MF 01"

1 st	2 nd
SEM	SEM
10:30-11:30	11:30-12:30
BEE	SOM

classmate

Date _____
Page _____



\therefore Linear strain = $\frac{\text{change in length}}{\text{original length}}$.

Lateral strain = $\frac{\text{change in diameter}}{\text{original diameter}}$.

\therefore Poisson ratio may be defined as the ratio of lateral strain to linear strain. Poisson ratio is denoted by (μ) .

$$\text{Poisson ratio} = \frac{\text{lateral strain}}{\text{linear strain}}$$

Ex

Q-1 A square steel rod 20mm x 20mm in section is to carry an axial load (compressive) of 100kN. Calculate the shortening in length of 500mm. $E = 2.14 \times 10^8 \text{ kN/m}^2$.

Sol: Given - (Area Export किने जाये Data को स.7. Units में लिखें)

$$\text{Area} = A = 20\text{mm} \times 20\text{mm} = 0.02\text{m} \times 0.02\text{m}$$

$$P = 100 \text{ kN} = 100 \times 10^3 \text{ N (compressive)}$$

$$L = 500\text{mm} = 50 \times 10^3 \text{ m}$$

$$E = 2.14 \times 10^8 \text{ kN/m}^2 = 2.14 \times 10^8 \times 10^3 \text{ N/m}^2$$

$$\delta l = ?$$

from the formula:

$$\delta l = \frac{P \times L}{A \times E} \quad (\text{rather val})$$

Putting the values:-

$$\delta l = \frac{100 \times 10^3 \times 50 \times 10^{-3}}{0.02 \times 0.02 \times 2.14 \times 10^8 \times 10^3}$$

$$\delta l = 0.584 \times 10^{-3} \text{ m} \quad \underline{\text{Answer}}$$

∴ As the given load is compressive in nature so the change in length i.e. shortening in length is -0.584×10^{-3} m.

Q:2 A hollow cast-iron cylinder 4m long, 300mm outer diameter, and thickness of metal 50mm is subjected to a central load on the top when standing straight. The stress produced is 75000 N/m^2 . Assume:- young's modulus = $1.5 \times 10^8 \text{ N/m}^2$.

- Find:- (i) magnitude of the load
(ii) longitudinal strain
(iii) total decrease in length.

Sol:- Given:-

$$L = 4 \text{ m}$$

$$D_o = 300 \text{ mm} = 0.3 \text{ m}$$

$$t = 50 \text{ mm} = 0.05 \text{ m}$$

$$\therefore D_i = D_o - 2t$$

$$= 0.3 - 0.05 \times 2$$

$$= 0.2 \text{ m}$$

D_o



Length = 4m = L

$$\text{Stress} = \sigma = 75000 \times 10^3 \text{ N/m}^2$$

$$D_i = 0.2 \text{ m} = D_o - 2t$$

$$E = 1.5 \times 10^8 \text{ N/m}^2$$

$$= 1.5 \times 10^8 \times 10^3 \text{ N/m}^2$$

$$= 1.5 \times 10^{11} \text{ N/m}^2$$

$$P = ?$$

$$E = ?$$

$$\delta l = ?$$

$$\therefore \sigma = \text{stress} = \frac{P}{A}$$

$$75000 \times 10^3 = \frac{P}{\frac{\pi}{4} \times (D_o^2 - D_i^2)}$$

$$\therefore P = 7500 \times 10^3 \times \frac{\pi}{4} \times (3^2 - 2^2)$$

$$P = 2945.2 \text{ kN. Ans}$$

again stress = $A \times \epsilon$ strain.

$$\therefore \text{strain} = \epsilon = \frac{\text{stress}}{E}$$

$$\therefore \epsilon = \frac{75000 \times 10^3}{1.5 \times 10^8 \times 10^3} = 0.0005 \text{ Ans}$$

$$\therefore \epsilon = 0.0005$$

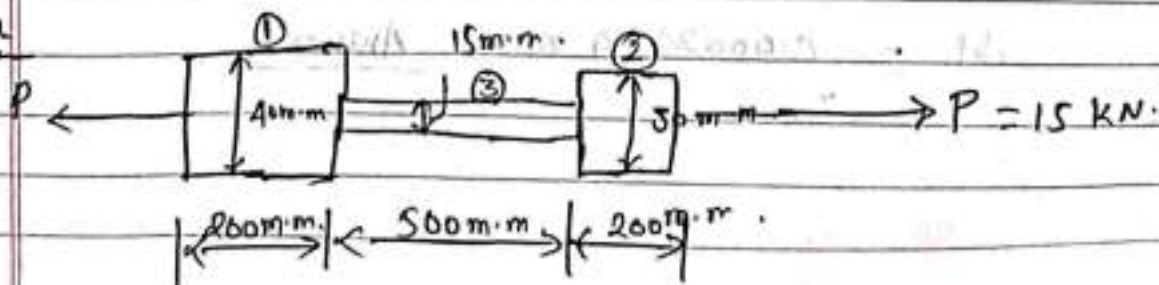
$$\text{and } \epsilon = \frac{\delta l}{l} \therefore \frac{\delta l}{l} = 0.0005$$

$$\delta l = 0.0005 \times l$$

$$\delta l = 0.002 \text{ m. Ans}$$

Q:3 A steel bar is 900 mm long, its two ends are 40 mm and 50 mm in diameter and the length of each rod is ~~200 mm~~ 200 mm. the middle portion of bar is 15 mm in diameter and 500 mm long. if the bar is subjected to an axial tensile load of 15 kN. find the total extension.
 $E = 200 \text{ GPa}$ $G \rightarrow \text{Giga.}$

Sol:



Given: $P = 15 \text{ kN} = 15 \times 10^3 \text{ N}$ $E = 200 \times 10^9 \text{ N/m}^2$.

$$\begin{aligned} D_1 &= 10 \text{ mm} = 0.01 \text{ m} & A_1 &= \frac{\pi}{4} \times 0.01^2 = 0.0000785 \text{ m}^2 \\ D_2 &= 20 \text{ mm} = 0.02 \text{ m} & A_2 &= \frac{\pi}{4} \times 0.02^2 = 0.000314 \text{ m}^2 \\ D_3 &= 15 \text{ mm} = 0.015 \text{ m} & A_3 &= \frac{\pi}{4} \times 0.015^2 = 0.0001767 \text{ m}^2 \end{aligned}$$

$$l_1 = 2 \text{ m} \quad l_2 = 2 \text{ m} \quad l_3 = 5 \text{ m}$$

Free body diagram:



$$\delta l_1 = \frac{P_1 l_1}{A_1 E_1} = \frac{15 \times 10^3 \times 2}{0.0000785 \times 200 \times 10^9} = 1.94 \times 10^{-5} \text{ m}$$

$$\delta l_2 = \frac{P_2 l_2}{A_2 E_2} = \frac{15 \times 10^3 \times 2}{0.000314 \times 200 \times 10^9} = 2.122 \times 10^{-5} \text{ m}$$

$$\delta l_3 = \frac{P_3 l_3}{A_3 E_3} = \frac{15 \times 10^3 \times 5}{0.0001767 \times 200 \times 10^9} = 2.122 \times 10^{-4} \text{ m}$$

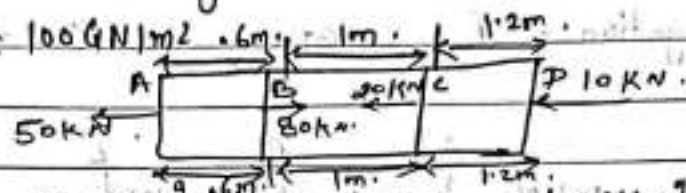
Total extension of bar:

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$\delta l = \left[\frac{15 \times 10^3 \times 2}{0.0000785 \times 200 \times 10^9} + \frac{15 \times 10^3 \times 2}{0.000314 \times 200 \times 10^9} + \frac{15 \times 10^3 \times 5}{0.0001767 \times 200 \times 10^9} \right]$$

$$\delta l = 0.0002454 \text{ m} \quad \text{Answer!}$$

Q:-4 A bar having cross-sectional Area 1000 mm^2 is subjected to axial force shown in fig. Find the total elongation of bar.



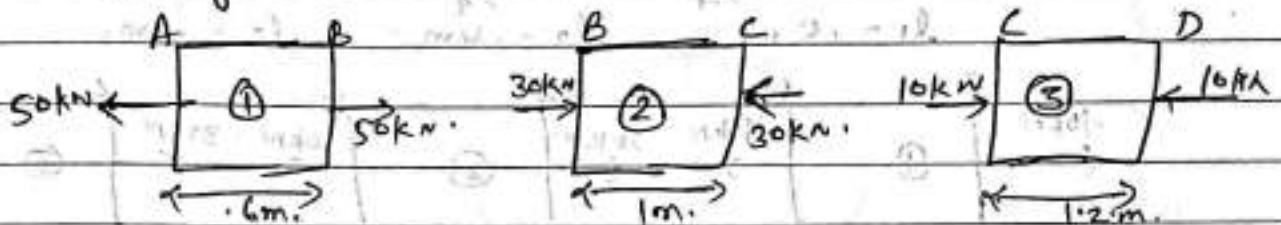
Sol:- Given:-

$$E = 100 \times 10^9 \text{ N/m}^2 = 100 \text{ GN/m}^2 = 100 \times 10^9 \text{ N/m}^2$$

$$A = 1000 \text{ mm}^2$$

$$= 1000 \times 10^{-6} \text{ m}^2 = 10^{-3} \text{ m}^2$$

Free body diagram:



$$\delta_1 = \frac{P_1 \times L_1}{A_1 \times E_1} \quad \delta_2 = \frac{P_2 \times L_2}{A_2 \times E_2} \quad \delta_3 = \frac{P_3 \times L_3}{A_3 \times E_3}$$

Sol

BC and CD body में force compressive है अर्थात् दोनों body में लम्बाई में कमी आयेगी इसलिए परिणामी लम्बाई में परिवर्तन -

$$\delta_{\text{net}} = \delta_1 - \delta_2 - \delta_3$$

$$\delta_{\text{net}} = \frac{50 \times 10^3 \times 0.6}{10^{-3} \times 10^{11}} - \frac{30 \times 10^3 \times 1}{10^{-3} \times 10^{11}} - \frac{10 \times 10^3 \times 1.2}{10^{-3} \times 10^{11}}$$

$$= -0.0012 \text{ m}$$

$$\delta_{\text{net}} = -0.12 \text{ mm} \quad \text{Answer}$$

Sol (-) negative sign shows that, बार की लम्बाई में -0.12 mm की कमी आयेगी।

$$\Delta l_{\text{net}} = \frac{P_1 l_1}{A_1 E} + \frac{P_2 l_2}{A_2 E} + \frac{P_3 l_3}{A_3 E}$$

$$= \frac{50 \times 10^3 \times 1}{600 \times 10^{-6} \times 210 \times 10^9} + \frac{3000 \times 10^3 \times 1}{2400 \times 10^{-6} \times 210 \times 10^9} + \frac{200 \times 10^3 \times 1}{1200 \times 10^{-6} \times 210 \times 10^9}$$

$\Delta l_{\text{net}} = 0.278 \times 10^{-3} \text{ m}$. So the length of the bar will increase by $= 0.278 \times 10^{-3} \text{ m}$.

Stress - strain curve for mild steel (Ductile) :-

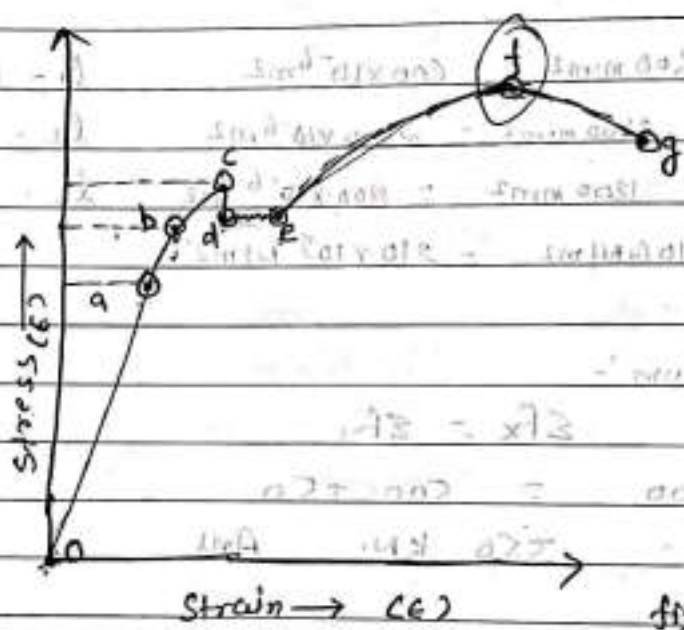


fig shows the typical stress - strain curve for ductile material (mild steel). Initially curve starts from point O where there is no stress and strain in specimen.

up to point a, Hooke's law obeyed where stress is directly proportional strain therefore, Oa is a straight line and point a is called the limit of proportionality and stress at point a is known as proportional limit stress.

* The portion of graph between a & b is not straight line but upto point b material remain elastic i.e. (on removal of load material regains its original shape). Point b is known as elastic limit point and the stress corresponding to that is called the elastic limit stress (σ_e).

In actual practice point a & b are so close to each other that it become difficult to differentiate between them.

* Beyond the point b material go to the plastic stage until upper yield point "c" is reached. At this point the cross-section area of material starts decreasing and stress decreases to a lower value to a point d, called lower yield point. Corresponding point c, the stress is known as upper yield point stress σ_{yu} , and corresponding to point d, the stress is known as ~~lower~~ ^{Lower} yield point stress, σ_{yl} .

* At point d, the specimen elongate by a considerable amount without any increase in stress and upto point e. the portion de is called yielding of the material at const. stress.

* From point e, the strain hardening phenomenon becomes predominant and the strength of the material increases, thereb requires more stress for deformation, until point f is reached. point f is known as ultimate point and stress corresponding to this stress is known as ultimate stress σ_u . it is the maximum stress to which material can be subjected in a simple tensile test.

* At point f the necking of the material begins and the cross-section area start decreasing at a rapid rate

Due to this local necking, the stress in material goes on decreasing in spite that the actual stress intensity goes on increasing. Ultimately the specimen breaks at point g , known as g breaking point corresponding stress known as breaking stress based upon the original cross section area.

Relation between elastic constant!

$$(i) \quad E = 2G(1+\mu)$$

$$(ii) \quad E = 3K(1-2\mu)$$

$$(iii) \quad E = \frac{9KG}{3K+G}$$

where E = Modulus of elasticity

G = Modulus of rigidity

K = bulk modulus.

μ = poisson ratio.

R.T.

Q: A concrete cylinder of diameter 150 mm and length 300 mm when subjected to an axial compressive load of 240 kN resulted in an increase of diameter by 0.127 mm and a decrease in length of 0.28 mm. Calculate the value of poisson ratio and modulus of elastic (E).

Sol: Given that:

$$d = 150 \text{ mm} = 0.15 \text{ m}$$

$$l = 300 \text{ mm} = 0.3 \text{ m}$$

$$P = 240 \text{ kN} = 240 \times 10^3 \text{ N}$$

$$\delta d = +0.127 \text{ mm} = +0.127 \times 10^{-3} \text{ m}$$

$$\delta l = -0.28 \text{ mm} = -0.28 \times 10^{-3} \text{ m}$$

$$\therefore \text{Linear strain} = \frac{\Delta l}{l} = \frac{.128 \times 10^{-3}}{.150} = .000933$$

$$\text{Lateral strain} = \frac{\Delta d}{d} = \frac{.127 \times 10^{-3}}{.150} = .000846$$

$$\therefore \text{Poisson ratio} = \frac{\text{lateral strain}}{\text{linear strain}}$$

$$= \frac{.000846}{.000933}$$

$$\mu = .907 \quad \underline{\text{Ans}}$$

$$\text{and } \Delta l = \frac{Pl}{AE}$$

$$E = \frac{Pl}{A \times \Delta l} \quad \text{on solving}$$

$$E = 14.55 \times 10^9 \text{ N/m}^2 \quad \underline{\text{Ans}}$$

Q: the following data related to a bar subjected to tensile test:

$d = 30 \text{ mm}$, Tensile load $P = 54 \text{ kN}$, Gauge length $l = 300 \text{ mm}$.

Extension of the bar $\Delta l = .122 \text{ mm}$, Change in dia, $\Delta d = .00366 \text{ mm}$

find: (i) poisson ratio (ii) the value of three moduli.

Sol:

$$\mu = \text{poisson ratio} = \frac{\text{lateral strain}}{\text{linear strain}}$$

$$= \frac{\Delta d/d}{\Delta l/l}$$

$$= \frac{.00366 \times 300}{.122 \times 10^{-3}} = .327 \quad \underline{\text{Ans}}$$

$$3.73 \times 10^4$$

$$4.066 \times 10^{-4}$$

and: $E = \frac{\text{Stress}}{\text{Strain}}$

$$= \frac{54 \times 10^3 / 1/4 \times 10^{-3}}{3.73 \times 10^{-4}}$$

$$= 2.05 \times 10^5 \text{ MN/m}^2. \text{ Ans}$$

$$\therefore E = 2G(1+\mu)$$

$$G = \frac{E}{2(1+\mu)} = 1.77 \times 10^5 \text{ MN/m}^2 \text{ Ans}$$

and

$$E = 3K(1-2\mu)$$

$$K = \frac{E}{3(1-2\mu)}$$

$$= 1.97 \times 10^5 \text{ MN/m}^2 \text{ Ans}$$

Thermal stress & strain:

Stress which induced in a body due to change in the temperature is known as thermal stress and the corresponding strain known as thermal strain.

- let l = length of a bar of uniform cross-sectional area
- t_1 = initial temperature of bar
- t_2 = final temperature
- α = coefficient of linear expansion.

So extension of the bar due to rise in temperature

$$\delta l = \alpha \times (t_2 - t_1) \times l.$$

When ~~the~~ ~~strain~~ it this elongation of the bar is prevented by some external forces. temperature strain will be given by:-

$$\text{thermal strain} = \frac{\Delta l}{l} = \alpha (t_2 - t_1) \times l$$

$$E_T = \alpha (t_2 - t_1) \cdot l$$

and

$$\text{thermal stress developed} = \alpha (t_2 - t_1) \times E$$

* when body is free to expand, then thermal stress will be zero.

* When temperature increases ^{then} thermal stress nature will be compressive.

* when temperature decreases then thermal stress nature will be tensile.

Strain Energy:- When an elastic body is loaded it undergoes deformation i.e. its dimensions change and when it is relieved of the load it regains its original shape. for the time of loading energy is stored in it, the same is released when load is removed. this energy is called strain energy.

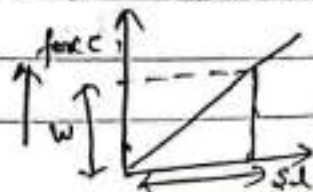
The strain energy stored by the body within elastic limit when loaded externally is known as 'resilience' and the maximum which a body stores up to elastic limit is called "Proof resilience."

(1) Due to gradually applied load:-

Strain energy stored in the bar = Work done by the load

$$U = \text{Avg. load} \times \text{distance.} \quad \text{force}$$

$$U = \frac{1}{2} \times W \times \Delta l$$



Q:- A steel bar 4cm. by 4cm in section, 3m. long is subjected to an axial pull of 128 kN. Taking $E = 200 \text{ GN/m}^2$ Calculate the alteration in the length of the bar. Calculate the amount of energy stored in the bar during extension.

Sol:- Given:-

$$\text{Area} = 4\text{cm} \times 4\text{cm} = 16\text{cm}^2$$

$$= 16 \times 10^{-4} \text{m}^2$$

$$W = 128 \text{ kN} \quad E = 200 \times 10^9 \text{ N/m}^2$$

$\delta l = ?$ energy stored during elongation i.e. strain energy $U =$
 $l = 3\text{m}.$

$$\delta l = \frac{W \times l}{A \times E}$$

$$= \frac{128 \times 10^3 \times 3}{16 \times 10^{-4} \times 200 \times 10^9} = 0.0012 \text{ m. Ans.}$$

Energy stored during Elongation i.e. $U = \frac{\sigma^2 \times A \times l}{2 \times E}$

$$U = \frac{(8 \times 10^7)^2 \times 16 \times 10^{-4} \times 3}{2 \times 200 \times 10^9} \quad \because \sigma = \frac{W}{A}$$

$$U = 76.8 \text{ N-m}$$

$$U = 76.8 \text{ J Ans}$$

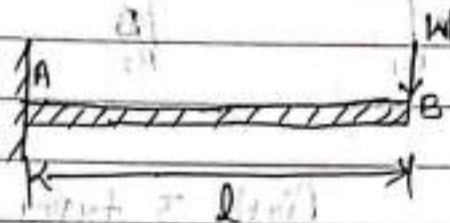
Unit - II

Shear forces and bending Moment

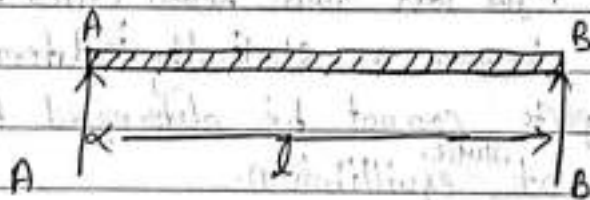
Beam:- the beam is defined as the structural member which is used to bear different loads. It resists vertical load, shear forces & bending moments.

Types of beam:-

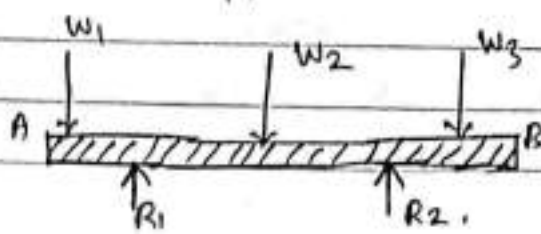
- (i) Cantilever beam:- A cantilever beam is a beam whose one end is fixed and other end is free.



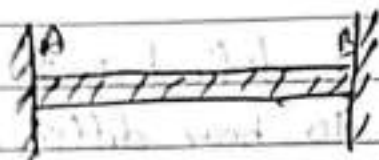
- (ii) Simply-supported beam:- A beam which is supported or resting on the support at its both the ends, is called simply supported beam.



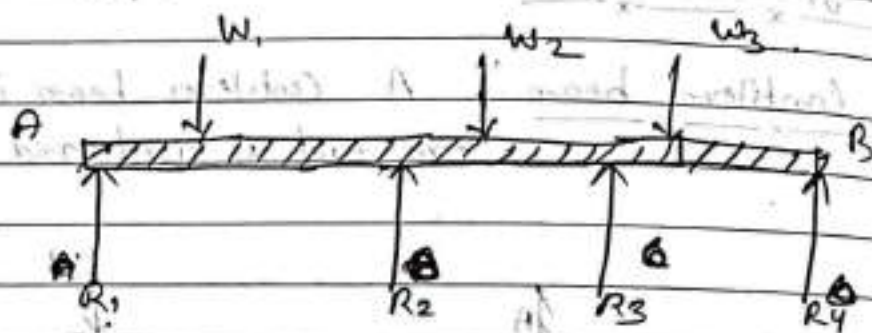
- (iii) Overhanging beam:- An overhanging beam is one in which the supports are not situated at the ends.



(iv) Fixed beam:- A beam which has both of its ends fixed or built in walls is called fixed beam.



(v) Continuous beam:- A beam which has more than two supports.



First 3 types of beam (cantilever, simply supported, overhanging beams) are subjected known as statically determinate beam. As the reaction of these beam at their support can be determine by the use of ~~static equilibrium~~ equation of static equilibrium.

The last two beam (fixed beam & continuous beam) are known as statically indeterminate beams as their supports cannot be determined by the use of equation of ^{static} equilibrium.

Shearing force: It is defined as the algebraic sum of forces acting either on left hand side or right hand side of section.

Unit - N. (Newton)

Bending Moment: - the moment produced by the forces acting on the beam must be ~~compared~~ balanced by equal and opposite moments produced by ~~the~~ the internal forces acting on the beam at the section. This is the bending moment at that section.

Unit - N-m.

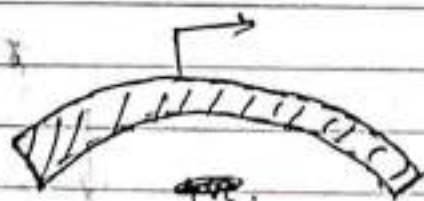
the bending moment which causes a beam to bend with the concave side upwards, is called a sagging bending moment, and it is taken as positive.



Sagging (+ve)

* in sagging type, the upper part is ^{under} compression stress and lower part is ~~under~~ under tensile stress.

the bending moment causing convexity upwards will be taken as negative and called Hogging bending moment.



Hogging (-ve)

* in hogging type the upper part is under tensile stress and lower part is under compressive stress.

Relation between shear force and bending moment-

$w = \frac{dF}{dx}$ where

w = intensity of load
F = shear force

and

$f = \frac{dM}{dx}$ where

f = shear stress
M = Bending moment

Q2

Point of contraflexure - In a beam if bending moment changes sign at a point, the point itself having zero bending moment, the 'changer curvature' at this point of zero bending moment and this point is called the point of contraflexure or point of inflexion or point of virtual hinge.

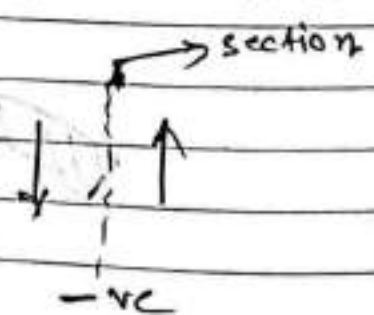
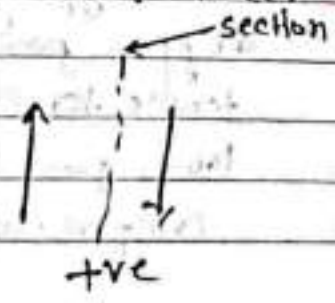
So at point of contraflexure.

$Bm = 0$

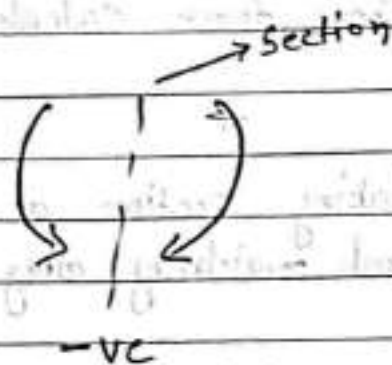
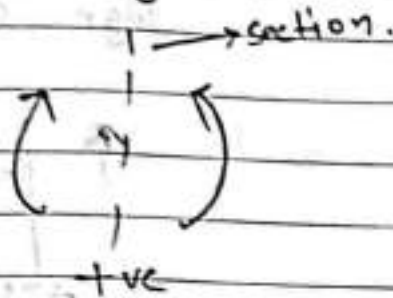
- * It is a point in bending moment diagram in which bending moment changes its sign from +ve or -ve to -ve or +ve
- * Point of contraflexure is that point in the beam which is not bending.

Sign Convention

for shear force:



for bending moment:-

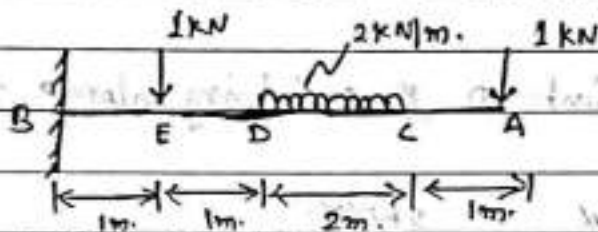


* this is the generally used sign convention. Some books may use different sign convention.

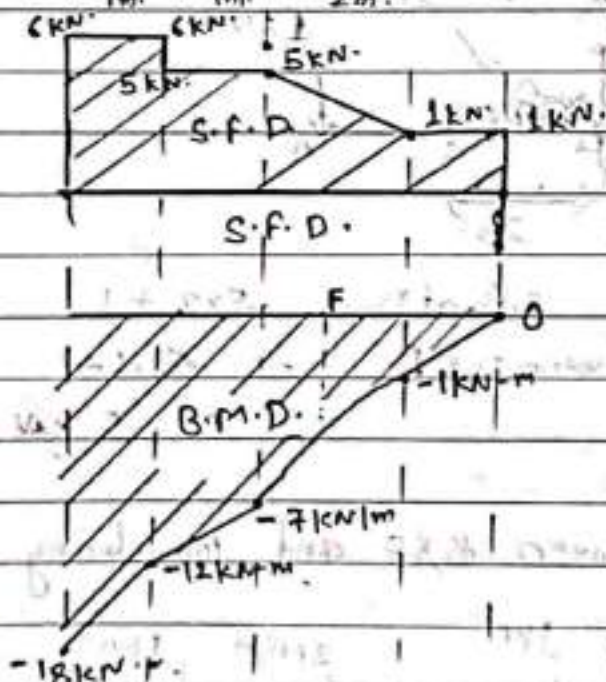
Numericals:-

(A) Cantilever beam:-

Q:-1 Draw the Shear force & Bending moment diagrams for cantilever beam loaded as shown in fig.



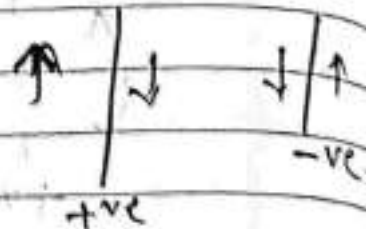
Sol:-



Shear force calculation:-

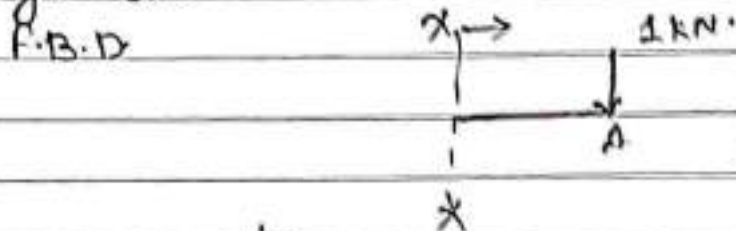
Sign convention

taking section at point A
and considering along right side.



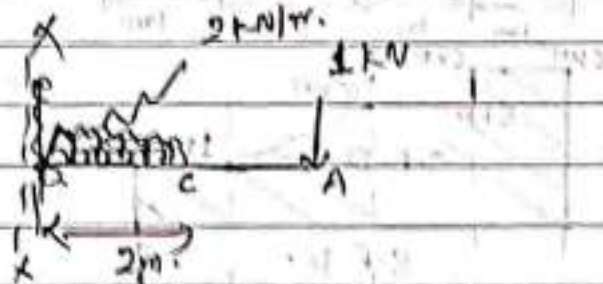
$$S.F. = 0 \text{ kN}$$

taking section between point C & A and considering
right side.



$$S.F. \text{ at } C = 1 \text{ kN}$$

taking section at point D & considering along right hand side.
F.B.D:-

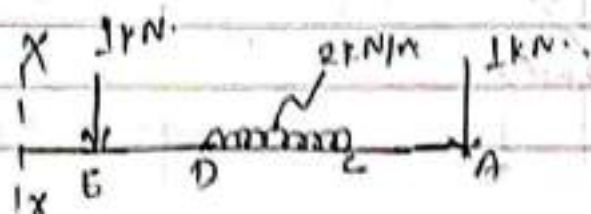


$$S.F. \Rightarrow S.F. \text{ at } D = 2 \times 2 + 1$$

$$S.F. \text{ between } D \& A = 5 \text{ kN}$$

$$= 5 \text{ kN}$$

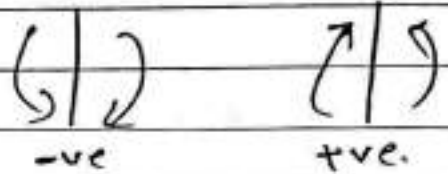
taking section between B & E and considering along right
hand side-



$$\begin{aligned} \text{S.F. between B \& C} &= 1 + 2 \times 2 + 1 \\ &= 6 \text{ kN} \end{aligned}$$

Bending moment calculation -

Sign convention.



At point A :-

taking section at point A and considering along R.H.S.

$$(B.M.)_A = 0$$

At point C ?

taking section at point C and considering along R.H.S. :-

$$(B.M.)_C = -1 \times CA$$

$$= -1 \times 1$$

$$(B.M.)_C = -1 \text{ kN}\cdot\text{m}$$

taking section at point D and considering along R.H.S.

$$B.M._D = -2 \times 2 \times 1 - 1 \times 3$$

$$= -4 - 3$$

$$= -7 \text{ kN}\cdot\text{m}$$

taking section at point E and considering along R.H.S.

$$(B.M.)_E = -2 \times 2 \times 2 \left(1 + \frac{2}{2}\right) - 1 \times 4$$

$$= -8 - 4 = -12 \text{ kN-m.}$$

taking section at B and considering R.H.S.:-

$$(B.M.)_B = -1 \times 1 - 2 \times 2 \left(1 + \frac{2}{2}\right) - 1 \times 5$$

$$= -1 - 12 - 5 = -18 \text{ kN-m.}$$

∴ portion betⁿ C/D is loaded with U.D.L. so the Bending moment diagram will be parabolic curve so we have to find a point in between to draw the curve.

So letⁿ consider a point F at the middle of C/D and

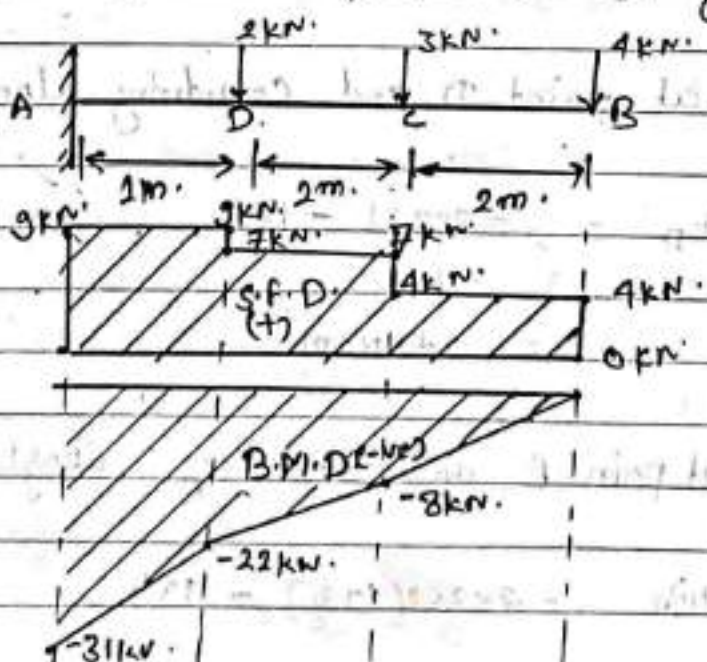
$$(B.M.)_F = -9 \times 1 \times \frac{1}{2} - 1 \times 2$$

$$= -3 \text{ kN-m.}$$

Q:-2

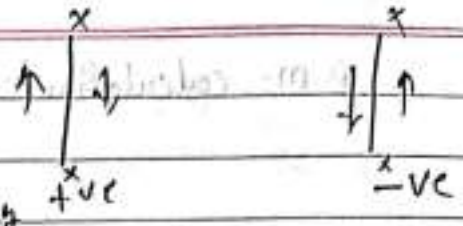
Draw the S.F.D. & B.M.D. of given beam:-

Sol:-



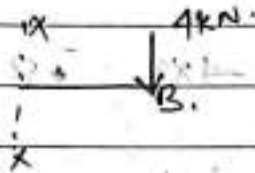
Shear force calculation:-

considering section at B & considering along right side.



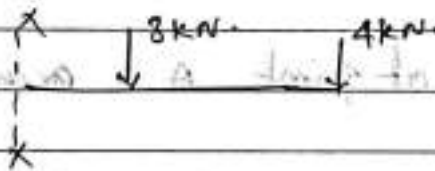
$$\text{S.F. at } B = 0 \text{ KN}$$

considering taking section between B & C and considering along R.H.S.



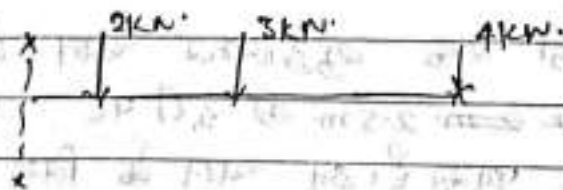
$$\text{S.F. between B \& C} = +4 \text{ KN}$$

taking section between C & D and considering along R.H.S.



$$\begin{aligned} \text{S.F. between C \& D} &= 4 + 3 \\ &= 7 \text{ KN} \end{aligned}$$

taking section between A & D and considering along R.H.S.

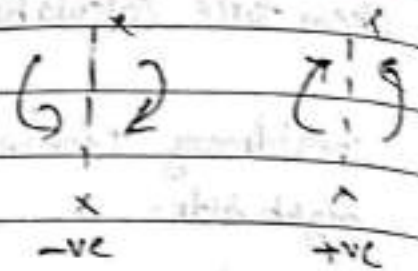


$$\begin{aligned} \text{S.F. bet}^n \text{ A \& D} &= 2 + 3 + 4 \\ &= 9 \text{ KN} \end{aligned}$$

B.M. calculation:-

taking section at B & considering along R.H.S:-

$$B.M._B = 0$$



taking section at point C & considering along R.H.S.

$$B.M._C = -4 \times 2 = -8 \text{ kN-m}$$

taking section at point D and considering along R.H.S.

$$B.M._D = -5 \times 2 + 4 \times 4$$

$$= -22 \text{ kN-m}$$

taking section at point A and considering along R.H.S.

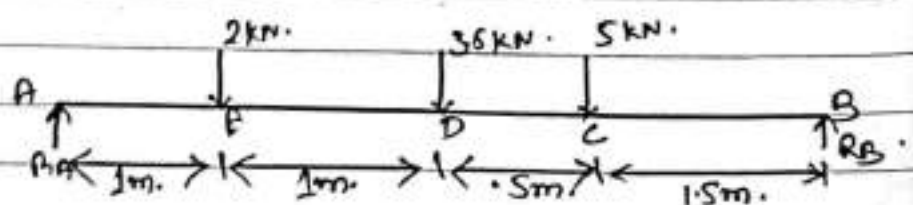
$$B.M._A = -2 \times 1 - 3 \times 3 - 4 \times 5$$

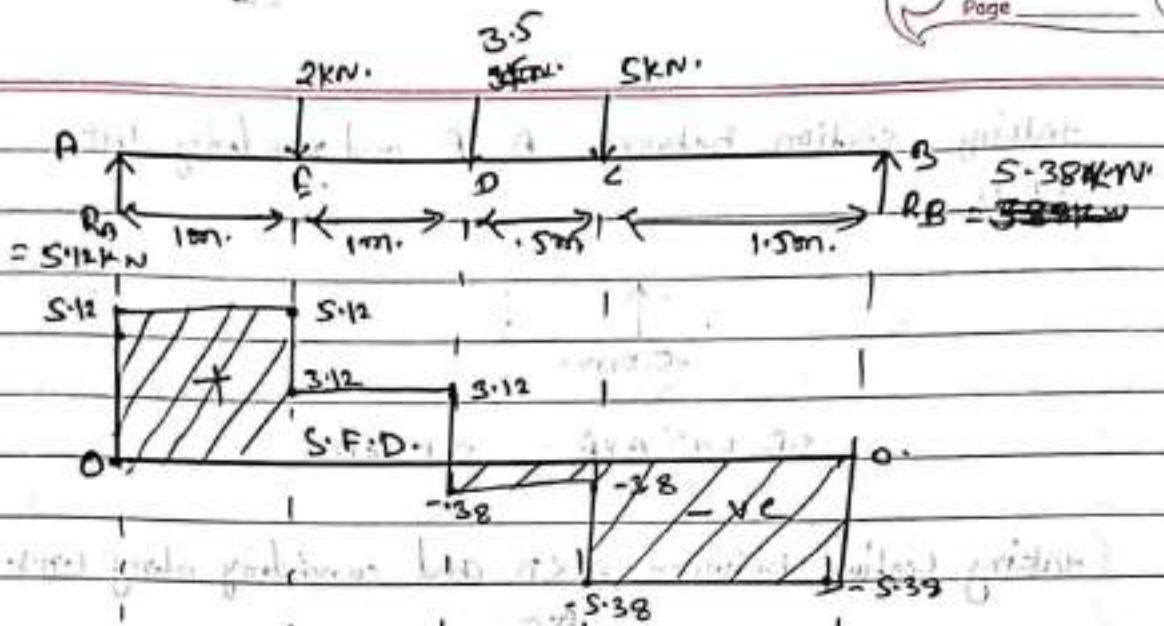
$$= -31 \text{ kN-m}$$

B. Simply supported Beam:-

Q. 1

4 m. लम्बाई की एक लुहलुंगल चरन के बांये आलरल से 1.2 m. और 2.5 m. की दूरी पर क्रमशः 2 kN, 3.5 kN, और 5 kN के भार प्रयुक्त हैं। इस चरन के लिए अपरुपण ललल अरुलर लल अलंकन आरुण अरुल बनाओ।





$$\sum F_y = 0$$

$$R_A + R_B = 2 + 38.5$$

$$R_A - R_B = -10.5 \text{ kN} \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$R_B \times 4 = 2 \times 1 + 3.5 \times 2 \times 2 + 5 \times 2.5$$

$$R_B = 5.38 \text{ kN}$$

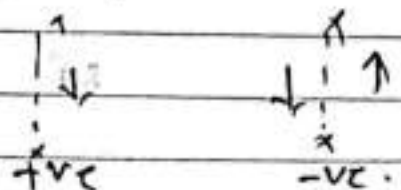
$$R_A = 17.12 \text{ kN}$$

Shear force calculation:-

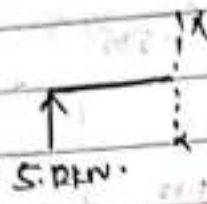
Sign convention

taking section at A and considering L.H.S.

$$S.F.A = 0$$

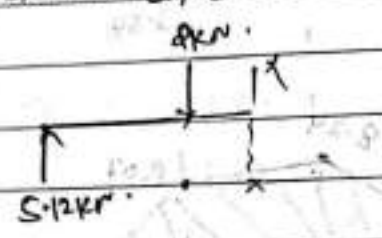


taking section between A & E and considering left side
F.B.D



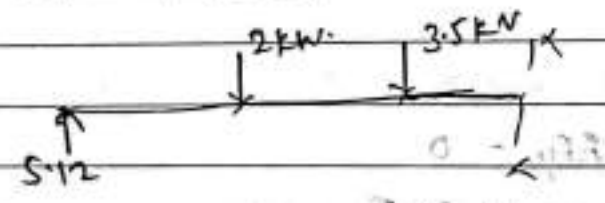
S.F. betⁿ A & E = 5.12 kN

taking section between E & D and considering along left side -



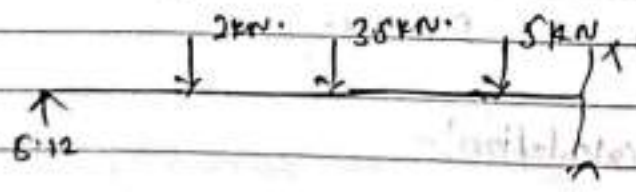
S.F. between E & D = 5.12 + 2 - 3.5 = 3.62 kN

taking section between D & C and considering left side.



S.F. betⁿ D & C = 5.12 + 2 - 3.5 - 4.9
= -3.08 kN

taking section betⁿ C & B and considering left side.

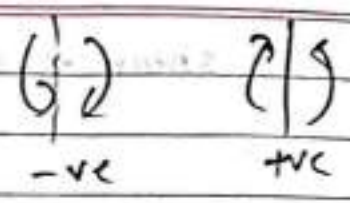


S.F. betⁿ C & B = 5.12 + 2 - 3.5 - 5
= -5.38 kN

taking section at B and considering right side =

S.F. B = 0

B.M. calculation.



B.M. at A = 0.

B.M. at E = $+5 \cdot 12 \times 1 = 5.12 \text{ kN}\cdot\text{m}$.

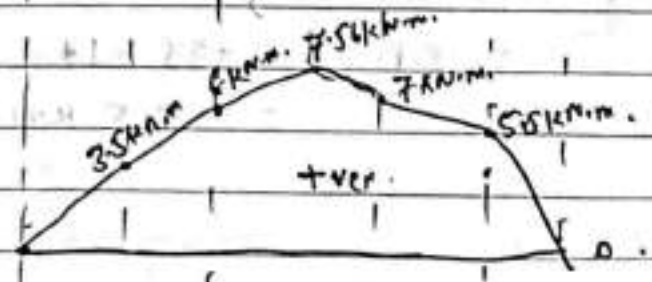
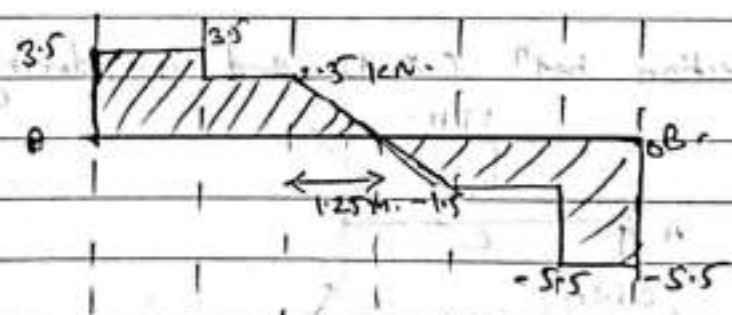
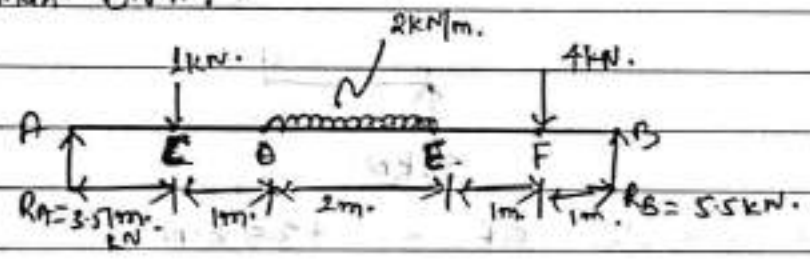
B.M. at D = $+5 \cdot 12 \times 2 = 120$
 $= 8.24 \text{ kN}\cdot\text{m}$.

B.M. at C = $+5 \cdot 12 \times 2.5 = 150$
 $= 18.07 \text{ kN}\cdot\text{m}$.

B.M. at B = 0.

Q:2 Draw the S.F. & B.M. diagram for the beam shown in fig. & find the max. B.M.

Sol:-



B.M. at taking section at C and considering L.H.S.

$$B.M.C = 3.5 \times 1 = 3.5 \text{ KN-m.}$$

taking section at D and considering L.H.S. =

$$B.M.D = 3.5 \times 2 - 1 \times 1$$

$$= 6 \text{ KN-m.}$$

taking section and at E and considering L.H.S. =

$$B.M.E = 3.5 \times 4 - 1 \times 3 - 2 \times 2 \times 1$$

$$= 7 \text{ KN-m.}$$

taking section at F and considering R.H.S. =

$$B.M.F = 5.5 \times 1$$

$$= 5.5 \text{ KN-m.}$$

Max. B.M. occur at the point where S.F. = 0.

So:

S.F. at D and E will be given by the eqⁿ

$$= 2.5 - 2x$$

$$\therefore 2.5 - 2x = 0$$

$$2x = 2.5$$

$$x = 1.25 \text{ m.}$$

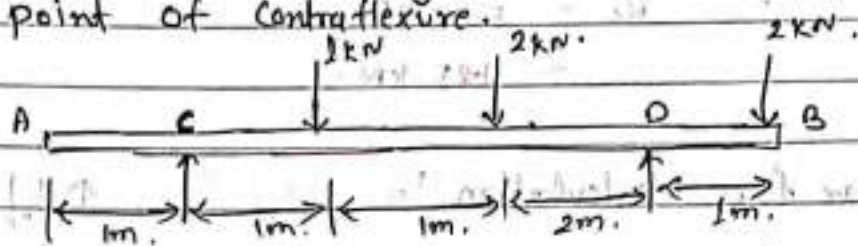
\(\therefore\) Max. B.M. will be produce at $1 + 1.25 \text{ m.}$
3.25m. from point A.

$$\therefore \text{M B.M.}_{\text{max}} = 3.5 \times 3.25 - 1 \times 2.25 - 2 \times 1.25 \times \frac{1.25}{2}$$

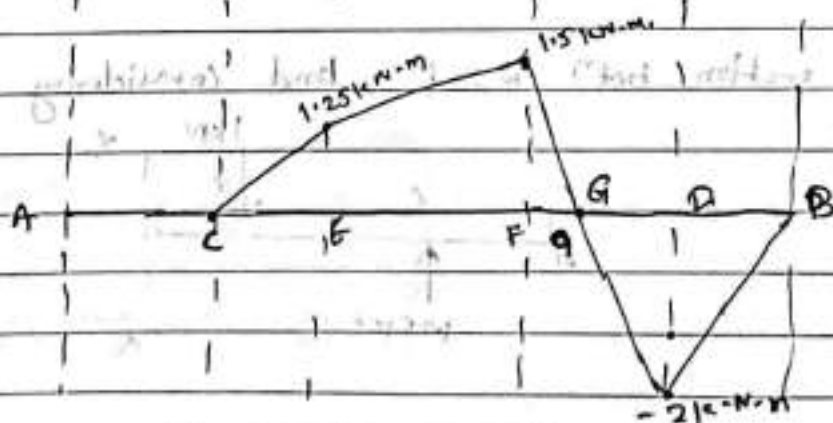
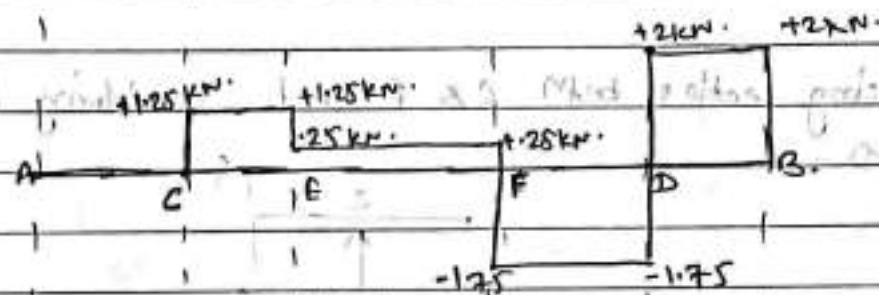
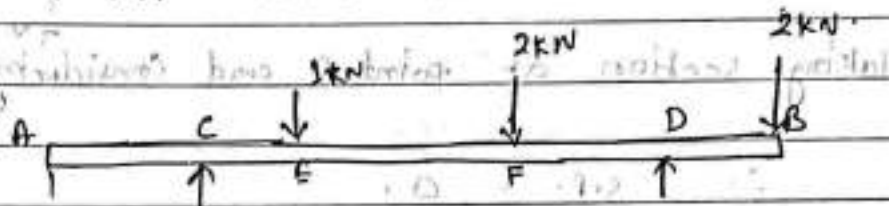
$$= 7.56 \text{ kN}\cdot\text{m.} \quad \text{Ans}$$

C. Over hanging beam:

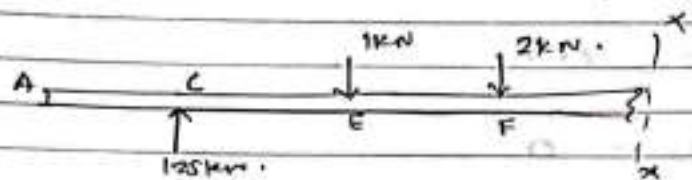
Q: 1 Draw the S.F. & B.M. diagram for given beam and find the point of contraflexure.



Sol:-



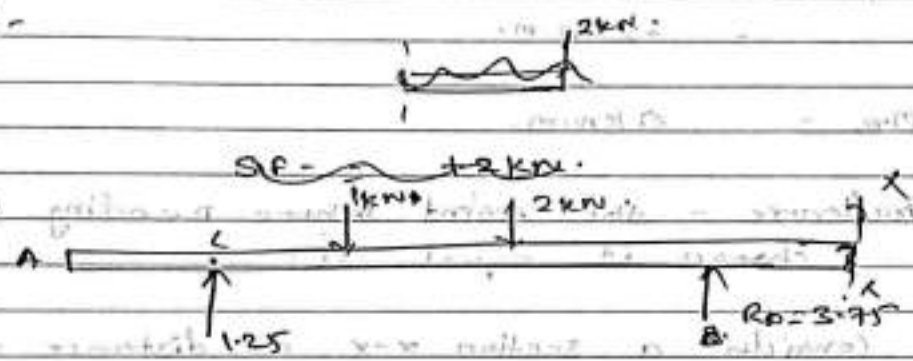
taking section betⁿ F & D and considering L.H.S. -
 F.B.D -



$$S.F = +1.25 - 1 - 2$$

$$= -1.75 \text{ kN}$$

taking section betⁿ D & B and considering R.H.S. -
 F.B.D :-



$$S.F = +1.25 - 1 - 2 + 3.75$$

$$= 2 \text{ kN}$$

taking section at B considering R.

Bending moment calculation:



taking section at A and considering it as

$$B.M_A = 0$$

and $B.M_C = 0$

$$B.M_E = + 1.25 \times 1 = 1.25 \text{ KN}\cdot\text{m}$$

$$B.M_F = + 1.25 \times 2 - 1 \times 1 = + 1.5 \text{ KN}\cdot\text{m}$$

$$B.M_D = - 2 \times 1 = - 2 \text{ KN}\cdot\text{m}$$

$$B.M_B = 0 \text{ KN}\cdot\text{m}$$

point of contraflexure - the point where bending moment changes its equal to 0.

Let's consider a section x-x at distance x from point B

$$B.M_x = + 3.75(x-1) - 2x^2$$

∴ at point of contraflexure, $B.M. = 0$

$$3.75x - 3.75 - 2x^2 = 0$$

$$1.75x = 3.75$$

$$x = \frac{3.75}{1.75}$$

$$x = 2.143 \text{ m}$$

Ans

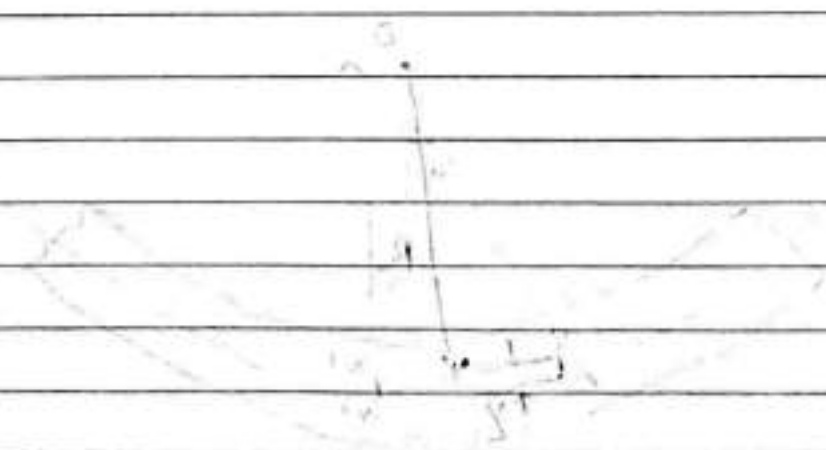
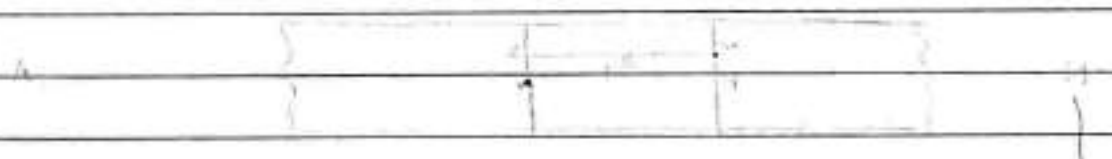
So point of contraflexure is situated at a distance 2.14m from point B.

(Faint handwritten text)

(Faint handwritten text)

(Faint handwritten text)

(Faint handwritten text, mostly illegible)



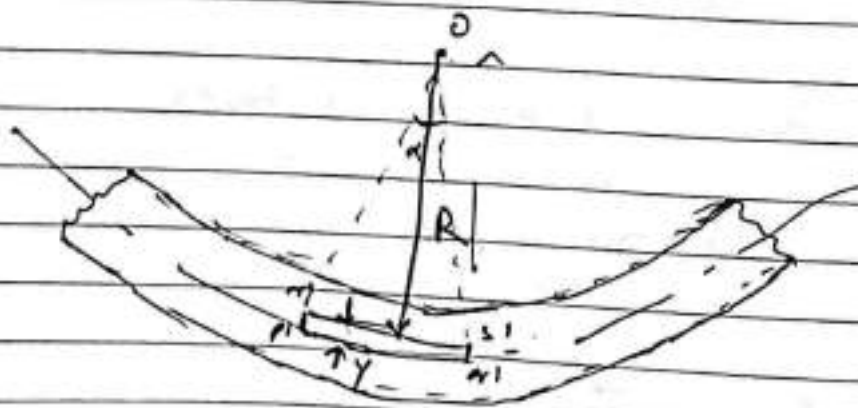
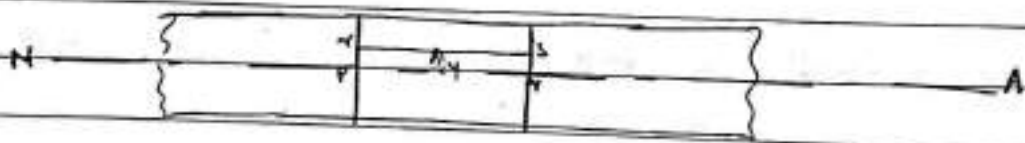
Unit - III

Bending stresses and shear stresses in beam type components.

Theory of simple bending:-

Assumption:-

1. The material of the beam is perfectly homogeneous throughout.
2. The stress is directly proportional to strain and at no place stress exceed the elastic limit.
3. The value of modulus of elasticity (E) is same, for the fibre of the beam under compression or ~~and~~ under tension.
4. There is no resultant pull or push on the cross-section of the beam.
5. The radius of curvature of the beam before bending is very ~~large~~ large in comparison to its transverse dimensions.



$$P'q' = R \alpha$$

$$r's' = (R-y) \alpha$$

Initially $Pq = rs$

and $Pq = P'q' \quad \therefore$ (No strain in N.A.)

$$\therefore \text{strain in } rs = \frac{rs - r's'}{rs} \quad \text{but } rs = Pq = P'q'$$

$$= \frac{P'q' - r's'}{rs} = \frac{R\alpha - (R-y)\alpha}{R\alpha}$$

$$= \frac{R\alpha - R\alpha + y\alpha}{R\alpha} = \frac{y}{R}$$

if stress in rs is σ and modulus of elasticity = E

$$\text{then strain in } rs = \frac{\sigma}{E}$$

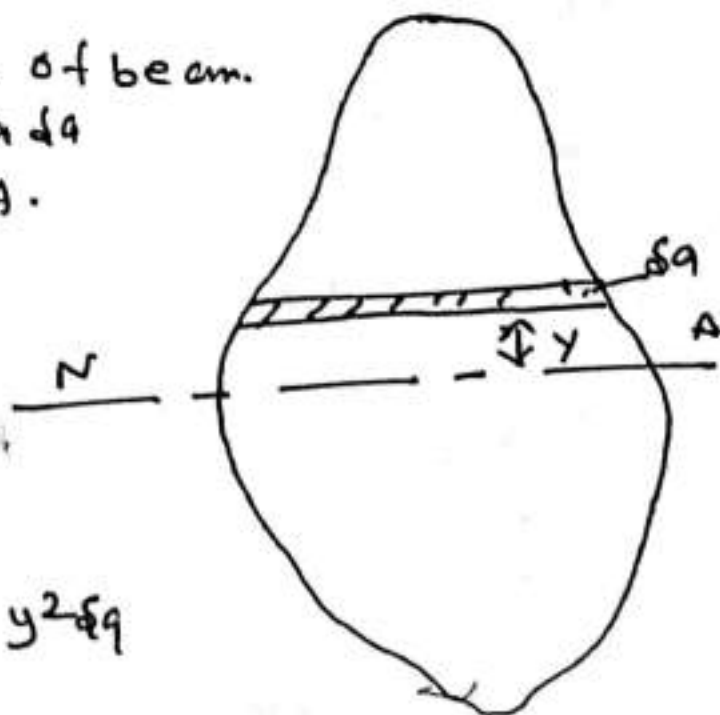
$$\therefore \frac{\sigma}{E} = \frac{y}{R}$$

$$\frac{\sigma}{y} = \frac{E}{R} \quad \text{--- (i)}$$

\therefore Consider a transverse section of beam.
and consider a strip of Area δa
at a distance y from N.A.

$$\therefore \text{Normal force on the Area} = \frac{E}{R} y \delta a$$

$$\text{Resisting moment} = \frac{E}{R} y^2 \delta a$$



and the total resisting moment is

$$= \sum \frac{E}{R} y^2 a \delta a$$

$$= \frac{E}{R} \sum y^2 \delta a$$

$\sum y^2 \delta a$ is the second moment of area about Neutral Axis

$$\therefore \text{Total Resisting Moment} = M = \frac{E}{R} \times I$$

$$\therefore \frac{M}{I} = \frac{E}{R}$$

from eq. 5.1

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$



Final equation is -

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where:

M = moment of resistance

I = Moment of inertia of the section about Neutral Axis (N.A.)

E = Young's Modulus of Elasticity

R = Radius of curvature of N.A.

σ = bending stress

Q.P.

* Neutral Axis or Neutral layer passes through the centre of area.

Q.P.

Section Modulus :

from equation $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I/y}$$

$$\sigma = \frac{M}{Z}$$

$$Z = \frac{I}{y}$$

Z = section modulus.

The strength of the beam depends mainly on the section modulus.

$$\text{Unit} = m^3.$$

Q. The section modulus for rectangular and circular sections are -

(i) Rectangular section:-

section modulus

(Z) = moment of inertia about neutral axis

distance of the most distant point from neutral axis



$$Z = \frac{I}{Y_{max}}$$

$$I = \frac{bd^3}{12}$$

$$\therefore Y_{max} = \frac{d}{2}$$

$$Z = \frac{bd^3}{12} \times \frac{2}{d}$$

$$Z = \frac{bd^2}{6}$$

(ii) circular section:-

$$Z = \frac{I}{Y_{max}}$$

$$I = \frac{\pi}{64} \times D^4$$

$$Y_{max} = \frac{D}{2}$$

$$Z = \frac{\pi}{64} \times D^4 \times \frac{2}{D}$$

$$\therefore Z = \frac{\pi}{32} \times D^3$$

Q:1 A 250 mm (depth) \times 150 mm (width) rectangular beam subject to maximum bending moment of 750 kN-m. Determine -

- (i) the max. stress in the beam
- (ii) if the value of E for the beam material is 200 GN/m^2 , find out the radius of curvature for that portion of the beam where the bending is maximum.

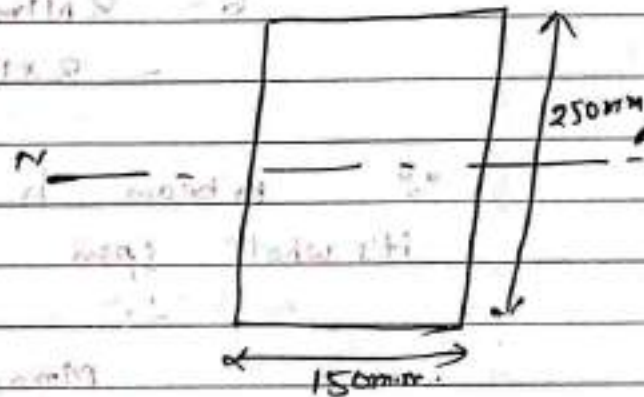
Find the value of longitudinal stress

Sol:

$$d = 250 \text{ mm}$$

$$b = 150 \text{ mm}$$

$$M = 750 \text{ kN-m} \\ = 750 \times 10^3 \text{ N-m}$$



$$I = \frac{b d^3}{12}$$

$$= \frac{15 \times 25^3}{12} = 0.001953 \text{ m}^4$$

$$y = \frac{d}{2} = \frac{250}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

$$\therefore \frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M \times y}{I}$$

$$= \frac{750 \times 10^3 \times 0.125}{0.001953} = 4.8 \times 10^8 \text{ N/m}^2$$

Ans

and $\frac{M}{I} = \frac{E}{R}$

$$R = \frac{E \times I}{M} = \frac{200 \times 10^9 \times 0.001953}{750 \times 10^3}$$

$$R = 52.08 \text{ m} \quad \text{Ans}$$

Q.1

4m. विस्तार (span) की एक आधातान्तर परिच्छेदी वृत्ताकार धारण पर 20 kN/m . का सगतिनरित भार (Udl) की धरी विस्तार पर प्रयुक्त है। गति का 2 की गहराई और चौड़ाई का अनुपात 1.5 हो तो अनुस प्रतिक्रम 8 N/m^2 के लिए गहराई और चौड़ाई का मान बताये।

sol:

$$l = 4 \text{ m.}$$

$$w = 20 \text{ kN/m.} \quad d/b = 1.5$$

$$G = 8 \text{ N/m}^2$$

$$= 8 \times 10^6 \text{ N/m}^2$$

∴ beam is simply supported and have Udl on its whole span

$$M_{\max} = \frac{wl^2}{8}$$

$$\frac{E I \sigma}{y}$$

$$= \frac{20 \times 4^2}{8} = 40 \text{ kN-m}$$

$$= 40 \times 10^3 \text{ N-m}$$

$$= 40 \times 10^3 \text{ N-m}$$

from bending equation:

$$\frac{M}{I} = \frac{G}{y}$$

$$\frac{40 \times 10^3}{I} = \frac{8 \times 10^6}{y}$$

$$\therefore \frac{I}{y} = \frac{40 \times 10^3}{8 \times 10^6}$$

$$\frac{b \times d^3}{12} \times \frac{2}{d} = \frac{40 \times 10^3}{8 \times 10^6}$$

$$\frac{bx^2}{6} = \frac{40 \times 10^3}{8 \times 10^6}$$

$\therefore d = 1.5 \times b$ put this value in above equation.

$$\frac{b \times (1.5 \times b)^2}{6} = \frac{40 \times 10^3}{8 \times 10^6}$$

$$b^3 = \frac{40 \times 10^3 \times 6}{1.5^2 \times 8 \times 10^6}$$

$$\therefore b = 0.237 \text{ m} \quad v = 0.24 \text{ m}$$

$$\therefore d = 1.5 \times 0.24 = 0.36 \text{ m}$$

Answer

Q.3 A symmetrical section 200 mm deep has a moment of inertia of $2.26 \times 10^{-5} \text{ m}^4$ about its neutral axis. Determine the longest span over which, when simply supported, the beam could carry a uniformly distributed load of 4 kN/m run without the stress due to bending exceeding 125 MN/m^2 .

Sol: $d = 200 \text{ mm}$

$$I = 2.26 \times 10^{-5} \text{ m}^4$$

$$l = ? \quad \text{UDL} = 4 \text{ kN/m}$$

$$\sigma = 125 \text{ MN/m}^2$$

In case of UDL in simply supported beam the max. bending moment is given by -

$$M = \frac{wl^2}{8}$$

$$M = \frac{4 \times l^2}{8}$$

from bending equation

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma \times I}{y}$$

$$M = \frac{125 \times 10^6 \times 2.26 \times 10^8}{250}$$

$$M = 28.25 \times 10^3 \text{ N.m.}$$

and

$$M = \frac{w l^2}{8}$$

$$28.25 \times 10^3 = \frac{4 \times 10^3 \times l^2}{8}$$

$$l^2 = \frac{28.25 \times 8}{4}$$

$$l = 7.516 \text{ m.} \quad \text{Ans}$$

Q:4 A rectangular 40mm. व्यास के एक ठोस, वृत्ताकार छेदी 3-तीर की लंबाई 2m. है। उसके मुक्त सिरे पर लगे अधिकतम बिन्दु भार का मान ज्ञात करो जिससे कि अपना दोग प्रबलन 50 N/mm^2 से अधिक न हो।

Sol:-

$$l = 2 \text{ m.} \quad \text{Beam - cantilever.}$$

$$\sigma = 50 \times 10^6 \text{ N/m}^2$$

let us assume load at the free end is $= w \text{ N.}$

$$\text{SO } M_{\text{max}} = w \times l = 2w \times \text{N.m.}$$

from bending equation -

$$\frac{M}{I} = \frac{\sigma}{y}$$

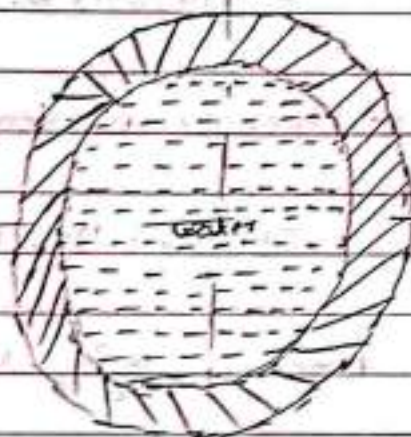
$$\frac{2 \times l}{\frac{\pi}{4} \times .04^4} = \frac{50 \times 10^6}{.02}$$

$$2 \times W = \frac{50 \times 10^6}{.02} \times \frac{\pi}{64} \times .04^4$$

$$W = 157 \text{ N.} \quad \underline{Ans}$$

Q:- A cast-iron water main 12m long of 500mm inside dia. and 25mm wall thickness runs full of water and is supported at its ends. Calculate the maximum stress in the metal if density of cast iron is 7200 kg/m^3 and that of water is 1000 kg/m^3 .

Sol:-



Given:-

$$l = 12 \text{ m.}$$

$$D_i = 500 \text{ mm} = .5 \text{ m.}$$

$$t = 25 \text{ mm} = .025 \text{ m.}$$

$$\rho_{ci} = 7200 \text{ kg/m}^3$$

$$\rho_w = 1000 \text{ kg/m}^3$$

$$\text{outer dia.} = D_o = D_i + 2t$$

$$= .5 + 2 \times .025$$

$$= .55 \text{ m.}$$

$$\text{cross-section area of main} = \frac{\pi}{4} (D_o^2 - D_i^2)$$

$$= \frac{\pi}{4} (.55^2 - .5^2)$$

$$= .04123 \text{ m}^2$$

weight of the water main in per meter length =

$$\rho = \frac{m}{V}$$

$$m = \rho \times V$$

$$m = \pi \rho \times \frac{\pi}{4} \times D^2 \times L$$

$$W = m \times g$$

$$W = 0.04123 \times 1 \times 7200 \times 9.81$$

$$= 2912.6 \text{ N}$$

Weight of water in one meter long main

$$W_w = \frac{\pi}{4} \times 0.5^2 \times 1 \times 1000 \times 9.81$$

$$= 1926.19 \text{ N}$$

Total weight of pipe per unit length meter length

$$= 2912.16 + 1926.19 = 4838.35 \text{ N}$$

$$\text{Max. B.M.} = M_{\text{max}} = \frac{wL^2}{8} = \frac{4838.35 \times 12^2}{8}$$

$$= 87090.3 \text{ Nm}$$

$$M.O.I. = I = \frac{\pi}{64} \times [(0.55)^4 - (0.5)^4] = 1.42384 \times 10^{-3} \text{ m}^4$$

$$y = \frac{D}{2} = \frac{0.55}{2} = 0.275 \text{ m}$$

from a bending eqn. $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma = \frac{M \times y}{I}$$

$$= \frac{87090.3 \times 0.275 \times 10^6}{1.42384 \times 10^3} = 16.82 \text{ MN/m}^2$$

Ans

Q: A rectangular strut is 20cm wide and 15cm thick. It carries a load of 60kN at an eccentricity of 2cm in a plane bisecting the thickness. Find the max. and min. intensities of stress in the section.

Sol: Given:-

$$\text{Width} = b = 20\text{cm} = 0.2\text{m}$$

$$\text{depth} = d = 15\text{cm} = 0.15\text{m}$$

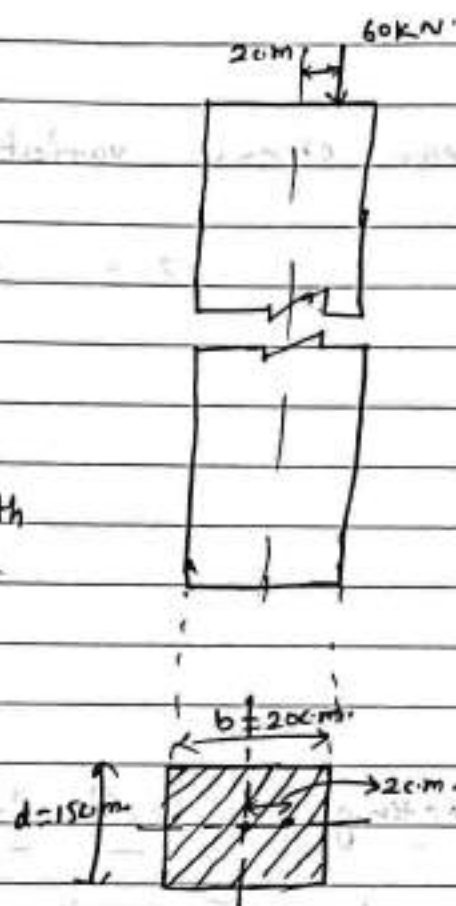
$$e = 2\text{cm} = 0.02\text{m}$$

$$W = 60\text{ kN}$$

\therefore load is not acting along the axis and have eccentricity of 2cm. So the will be 2 types of stress on work-

(i) direct stress

(ii) bending stress.



$$\text{direct stress} = \frac{W}{A} = \frac{60}{0.2 \times 0.15} = 2\text{ MN/m}^2$$

$$\text{bending stress} = \sigma_b = \frac{M}{Z}$$

$$= \frac{W \times e}{\frac{b \times d^3}{12} \times 2}$$

$$= \frac{60 \times 0.02}{\frac{0.15 \times 0.2^3}{12} \times 2} = 1.2\text{ MN/m}^2$$

$$\therefore \sigma_b = 1.2\text{ MN/m}^2$$

$$\therefore \text{Max. stress} = Gd + Gb = 2 + 1.2 = 3.2 \text{ MN/m}^2$$

$$\text{Min stress} = Gd - Gb = 2 - 1.2 = 0.8 \text{ MN/m}^2$$

Shear stress variation :-

$$Z = \frac{S A \bar{y}}{I b}$$

where:-

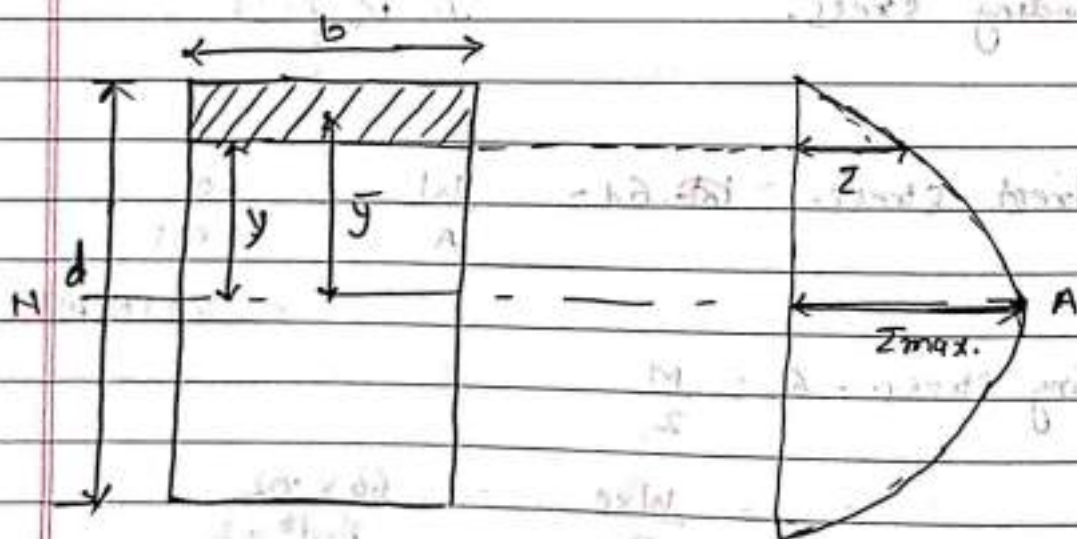
Z = shear stress, S = shear force

A = Area of section where shear stress is to be calculated.

\bar{y} = distance from N.A. to the centroid of the area.

I = M.O.I. of given fig. about N.A.

(i) Rectangular section:-



let Z be the shear stress in a layer at a distance y from Neutral Axis.

$$\therefore Z = \frac{S A \bar{y}}{I b}$$

$$A = \text{Area of shaded portion}$$

$$= b \left(\frac{d}{2} - y \right)$$

$$\text{and } \bar{y} = \frac{\left(\frac{d}{2} - y \right) + y}{2}$$

$$= \frac{\frac{d}{2} - y}{2} + y$$

$$= \frac{\frac{d}{2} + y}{2} = \frac{1}{2} \left(\frac{d}{2} + y \right)$$

$$I = \frac{b \times d^3}{12}$$

put value in formula.

$$Z = \frac{S \times b \left(\frac{d}{2} - y \right) \times \frac{1}{2} \left(\frac{d}{2} + y \right)}{\frac{b \times d^3}{12} \times b}$$

$$= \frac{12 \times S \times b}{b \times d^3 \times b} \times \left(\frac{d^2}{4} - y^2 \right)$$

$$= \frac{6S}{bd^3} \times \left(\frac{d^2}{4} - y^2 \right)$$

at $y = 0$

$$Z = Z_{\text{max}}$$

$$\therefore Z_{\text{max}} = \frac{6S}{bd^3} \times \frac{d^2}{4} = \frac{3S}{2bd}$$

$\frac{S}{bd}$ = mean shear stress (Z_{mean}).

$$\therefore \tau_{max} = \frac{3}{2} \tau_{mean}$$

(iii) Solid circular section:-

for circular section:

$$\tau = \frac{4}{3} \frac{S}{\pi R^4} (R^2 - y^2)$$

at $y=0$ τ will be τ_{max} .

$$\therefore \tau_{max} = \frac{4}{3} \frac{S}{\pi R^4} \times R^2$$

$$= \frac{4}{3} \frac{S}{\pi R^2}$$

$\frac{S}{\pi R^2}$ will be taken as mean shear stress (τ_{mean}).

$$\therefore \tau_{max} = \frac{4}{3} \tau_{mean}$$

Q:- A circular beam 150 mm diameter is subjected to a shear stress force of 7 kN. calculate the value of max. shear stress, and sketch the variation of shear stress along the depth of the beam.

Sol:- Given that:-

$$d = 150 \text{ mm} = 0.15 \text{ m}$$

$$S = 7 \text{ kN} = 7000 \text{ N}$$

$$\tau_{mean} = \frac{S}{A} = \frac{S}{\pi/4 \times d^2}$$

$$= \frac{7 \times 10^3}{\pi/4 \times 15^2} = 396 \text{ KN/m}^2.$$

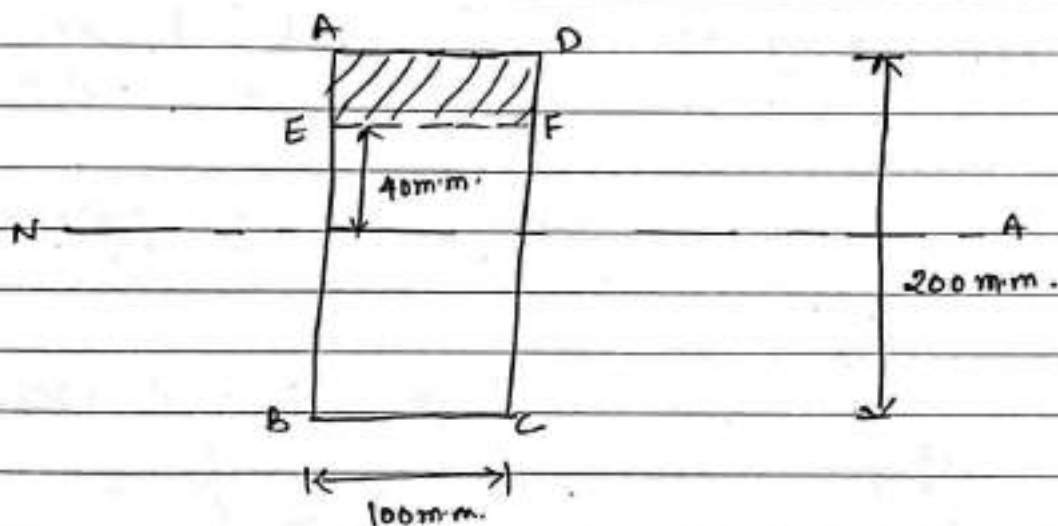
for Circular Section

$$\tau_{max} = \frac{4}{3} \tau_{mean}$$

$$\tau_{max} = \frac{4}{3} \times 396 = 528 \text{ KN/m}^2. \quad \underline{\text{Ans}}$$

Q:2 एक प्रकीर्ण की आयताकार धारण 100mm चौड़ा और 200mm गहरी है और दोनो सिरो में खुलात्मकित है। धारण के मध्यबिन्दु पर एक 20KN का भार प्रयुक्त है। धारण की एक उदासीन अक्ष से 40mm की दूरी स्थित बिन्दु पर अपरतण प्रतिबल कात करो।

sol:-



$$\tau = \frac{S \times A \times \bar{y}}{I \times b}$$

$$\text{Max. Shear force} = \frac{W}{2} = \frac{20}{2} = 10 \text{ KN.}$$

$$A = \text{Area of ADEF} = 100 \times 60 = 6000 \text{ mm}^2 = 6000 \times 10^{-6} \text{ m}^2$$

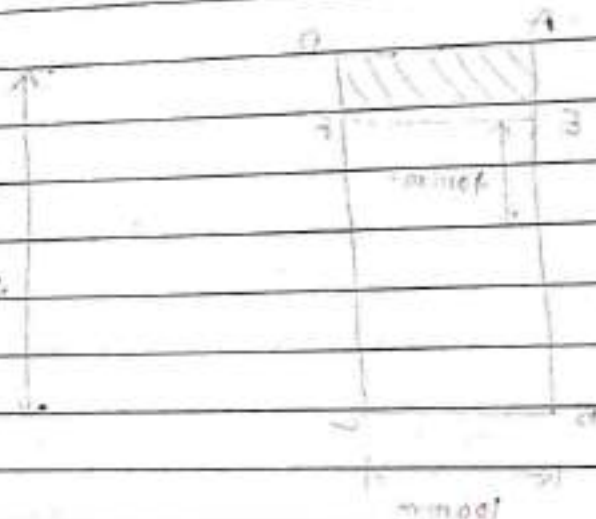
and $\bar{y} = \frac{40 + 60}{2} = 70 \text{ mm} = 70 \times 10^{-3} \text{ m}$

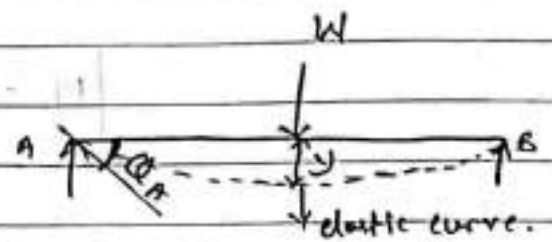
$$I = \frac{b d^3}{12} = \frac{1 \times 23^3}{12}$$

put all the value in formula.

$$Z = \frac{1 \times 10^3 \times 6000 \times 10^{-6} \times 70 \times 10^{-3}}{\frac{1 \times 23^3}{12}}$$

$$Z = 63 \times 10^6 \text{ N/m}^2 \quad \text{Ans}$$



Unit - 4Deflection of beam type components.Deflection :-

It is the vertical distance of the beam measured before and after loading.

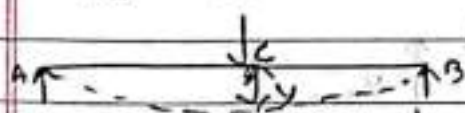
- * It is denoted by y .
- * Deflection at the supports is always zero.

Slope :- It is the angle in radian measured betⁿ the tangent to the elastic curve & the original of the beam.

- * Slope is denoted by θ or $\frac{dy}{dx}$.
- * It's unit is radians.

Boundary conditions :-

(i) Simply supported beam



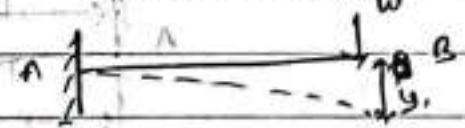
* At A & B deflection = 0

* At C deflection = max.

* At C slope is = Zero.

* At A & B slope is = max.

(ii) Cantilever beam



* At A deflection = 0

* At B deflection = max.

* At A slope = 0

* At B slope = max.

Relation betⁿ slope, deflection & radius of curvature:-

$$M = EI \times \frac{d^2y}{dx^2}$$

the equation is based on Bending moment only. the effect of shear force is neglected.

where
 M = Bending moment
 I = Moment of Inertia
 E = Modulus of Elasticity
 y = deflection
 $\frac{dy}{dx}$ = slope

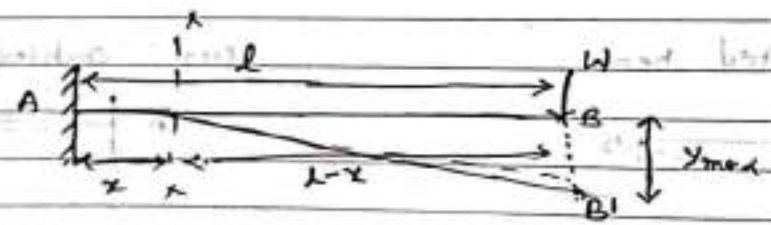
The isop methods used for finding out the slope & deflection:-

- (i) Double integration method
- (ii) Moment Area method
- (iii) Macaulay's Method.

Cantilever beam:-

1 pt

Cantilever beam with a point load W at free end



A cantilever beam AB length l and fixed at point A, having a point load W at free end B.

Let consider a section x-y at a distance x from fixed end. So Bending moment at section x-y -

$$M_x = -wl(1-x) \quad \text{--- (i)} \quad \begin{matrix} P/3 & G/2 \\ +ve & -ve \end{matrix}$$

$\therefore EI \frac{d^2y}{dx^2} = M_x$ So eq (i) can be written as:

as:

$$EI \frac{d^2y}{dx^2} = -wl(1-x)$$

Integrating both sides

$$EI \frac{dy}{dx} = -wl \left(1x - \frac{x^2}{2} \right) + C_1 \quad \text{--- (ii)}$$

\therefore In case of cantilever beam at support slope $\frac{dy}{dx}$ will be equal to 0.

at $x=0$ $\frac{dy}{dx} = 0$ put the value in eq (ii)

$$C_1 = 0$$

$$EI \frac{dy}{dx} = -wl \left(1x - \frac{x^2}{2} \right) \quad \text{--- (iii)}$$

\therefore Slope at B = put $x=1$.

$$EI \frac{dy}{dx} = -wl \left(1^2 - \frac{1^2}{2} \right)$$

$$= \frac{-wl^2}{2}$$

$$\therefore \boxed{\frac{dy}{dx} = \theta_B = \frac{-wl^2}{2EI}}$$

again integrating eq. (iii) both side

$$EIy = -w \left(\frac{1}{2}x^2 - \frac{x^3}{6} \right) + C_2$$

in case of cantilever beam at $x=0$ deflection $y=0$

$$C_2 = 0$$

Put the value in above eqn.

$$EIy = -w \left(\frac{1}{2}x^2 - \frac{x^3}{6} \right) + 0$$

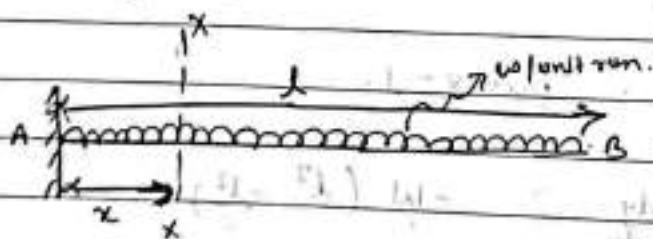
So deflection at point B will be put $x=l$

$$EIy = -w \left(\frac{l^3}{2} - \frac{l^3}{6} \right)$$

$$y_b = \frac{-wl^3}{3EI}$$

deflection at free end
Ans

II Ques
Cantilever beam having length l carrying w per unit length over whole length?



considering a cantilever beam of length l carrying a w of w per unit length over whole length.

let take a section $x-x$ from with distance x from fixed end.

So Bending moment along the section

$x-x$ will be -

$$M_x = -w \frac{(l-x) \cdot x(l-x)}{2}$$

$$\therefore M_x = -\frac{w(l-x)^2}{2}$$

$$\therefore M_x = EI \frac{d^2y}{dx^2}$$

So the above equation will

be written as -

$$EI \frac{d^2y}{dx^2} = -\frac{w(l-x)^2}{2} \quad \text{--- (i)}$$

Integrating both side.

$$EI \frac{dy}{dx} = +\frac{w(l-x)^3}{6} + C_1$$

for cantilever beam slope at fixed end will be zero.

$$\text{at } x=0 \quad \frac{dy}{dx} = 0$$

\therefore

$$0 = \frac{w(l-0)^3}{6} + C_1$$

$$C_1 = -\frac{wl^3}{6} \quad \text{put this value in above}$$

equation and

$$EI \frac{dy}{dx} = \frac{w(l-x)^3}{6} - \frac{wl^3}{6} \quad \text{--- (ii)}$$

So slope at free end will be

at put $x=l$.

$$\therefore EI \frac{dy}{dx} = 0 - \frac{wl^3}{6}$$

$$\left(\frac{dy}{dx}\right)_{at x=2} = 0_B = -\frac{wl^3}{6EI}$$

again integrating equation (1)

$$EIy = -\frac{wl}{6EI} (1-x)^4 - \frac{wl^3x}{6} + C_2$$

at $x=0$ (fixed end) deflection $y=0$.

$$\therefore 0 = -\frac{wl^4}{24} - \frac{wl^3 \cdot 0}{6} + C_2$$

$$\therefore C_2 = \frac{wl^4}{24} \quad \text{put the value in}$$

above eq.

$$EIy = -\frac{wl}{24} (1-x)^4 - \frac{wl^3x}{6} + \frac{wl^4}{24}$$

deflection at free end -

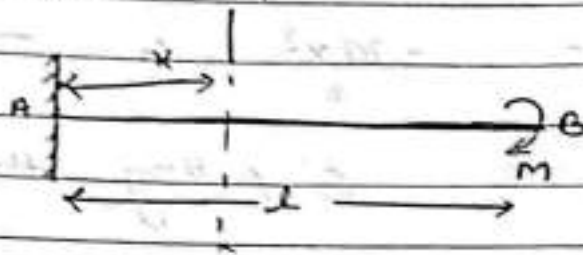
put $x=2$

$$\begin{aligned} EIy &= 0 - \frac{wl^4}{6} + \frac{wl^4}{24} \\ &= -\frac{wl^4}{8} \end{aligned}$$

$$\therefore \left(\frac{y}{l}\right)_B = -\frac{wl^4}{8EI}$$

deflection at free end will be given by this equation.

III Cantilever of length l with moment applied at the free end:



A cantilever beam AB of length l and fixed at point A and having a moment M at free end B.

So taking section $x-x$ at a distance x from fixed ~~base~~ ~~at~~ ~~end~~ and taking moment along ~~at~~ ~~at~~ $x-x$

$$M_x = -M$$

$$\therefore M_x = EI \frac{d^2y}{dx^2}$$

$$-M = EI \frac{d^2y}{dx^2}$$

on integrating both side.

$$-Mx + C_1 = EI \frac{dy}{dx} \quad \text{--- (1)}$$

putting

at $x=0$ $\frac{dy}{dx} = 0$ putting this value in eq. (1)

$$\therefore 0 + C_1 = 0$$

$$C_1 = 0$$

$$\therefore -Mx = EI \frac{dy}{dx}$$

again integrating both side.

$$-\frac{Mx^2}{2} + C_2 = EIy$$

$$EIy = -\frac{Mx^2}{2} + C_2 \quad \text{--- (1)}$$

at $x=0$ $y=0$ ∴ putting this value in eq (1)

$$0 = 0 + C_2$$

$$C_2 = 0$$

$$\therefore EIy = -\frac{Mx^2}{2}$$

$$\therefore \boxed{y = \frac{-Mx^2}{2EI}}$$

∴ deflection at free end ∴ at $x=l$.

$$\boxed{y = \frac{-MR^2}{2EI}}$$

Case iv Cantilever of length l carrying a point load W at distance a from the fixed end:-



\therefore Slope:-

$$\theta_B = \theta_C = \frac{-W a^2}{2EI}$$

and deflection will be given by:-

$$y_C = \frac{-W a^3}{3EI}$$

deflection at free end:-

$$y_D = y_C + (l-a) \times \theta_B$$

$$= \frac{-W a^3}{3EI} + (l-a) \times \frac{-W a^2}{2EI}$$

$$y_D = \frac{-W a^3}{3EI} - (l-a) \frac{W a^2}{2EI}$$

Numericals:-

Q: 1
100 mm चौड़ी और 200 mm गहरी एक लंबी-उत्तोलक की लंबाई 1.5 m है। उसके आबड़ सिरे से 1 m की दूरी पर एक 10 kN का बिन्दु भार लगा है। मुक्त सिरे पर शून्य एवं विक्षेप जात करो।

$$E = 2 \times 10^5 \text{ N/mm}^2$$

Sol: Given:

$$b = 100 \text{ mm} = 0.1 \text{ m}$$

$$d = 200 \text{ mm} = 0.2 \text{ m}$$

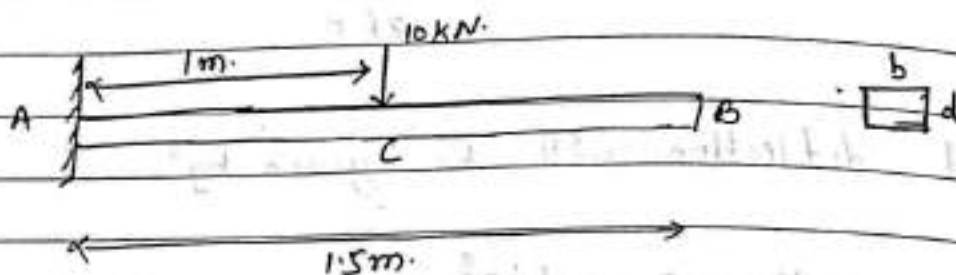
$$l = 1.5 \text{ m}$$

$$a = 1 \text{ m}$$

$$W = 10 \text{ kN} = 10 \times 10^3 \text{ N}$$

$$E = 2 \times 10^5 \text{ N/mm}^2 = 2 \times 10^5 \times 10^6 \text{ N/m}^2$$

$$= 2 \times 10^{11} \text{ N/m}^2.$$



$$\therefore \theta_B = ?$$

$$y_B = ?$$

$$\theta_B = \frac{-Wl^2}{2EI}$$

$$I = \frac{b \times d^3}{12} = \frac{.1 \times .2^3}{12}$$

$$= 66.67 \times 10^{-6} \text{ m}^4$$

$$\theta_B = - \frac{10 \times 10^3 \times 1^2}{2 \times 2 \times 10^{11} \times 66.67 \times 10^{-6}}$$

$$= - \frac{10 \times 10^3 \times 1^2}{2 \times 2 \times 10^{11} \times 66.67 \times 10^{-6}}$$

$$= .00375 \text{ radian.}$$

and

$$y_B = y_C + (l-a) \theta_B$$

$$= \frac{-Wl^3}{3EI} - (l-a) \frac{Wa^2}{2EI}$$

$$= - \frac{10 \times 10^3 \times 1^3}{3 \times 2 \times 10^{11} \times 66.67 \times 10^{-6}} - (1.5-1) \times \frac{10 \times 10^3 \times 1^2}{2 \times 2 \times 10^{11} \times 66.67 \times 10^{-6}}$$

$$y_C = - .4375 \text{ m/m}$$

\rightarrow -ve sign shows the downward deflection.

$$\therefore y_c = -4575 \text{ mm (downward) Ans}$$

Q.2. एक लंबा बरतन काट की ओर 3m लंबाई तक बेंधी इलाकिक बल पर के बिंदु पर बल है, का उपर का बल 20N का आठ मीरे से 2m की जां और दूसरा बल 10N का उठ मीरे पर है कुल मीरे पर बल का विद्युत जां को।

$$E = 2 \times 10^5 \text{ N/m}^2 \text{ and } I = \frac{550}{6} \times 10^4 \text{ mm}^4$$

Sol. Given:-

$$L = 3 \text{ m}$$

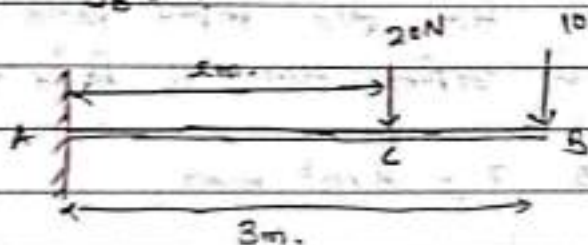
$$W_1 = 20 \text{ N}$$

$$W_2 = 10 \text{ N}$$

$$E = 2 \times 10^5 \text{ N/m}^2 = 2 \times 10^{11} \text{ N/m}^2$$

$$I = \frac{550}{6} \times 10^4 \text{ mm}^4 = \frac{550}{6} \times 10^{-8} \text{ m}^4$$

$$y_B = ?$$



at free end

deflection at free end = Deflection due to 20N + deflection at free end due to 10N.

$$= y_1 + y_2$$

$$\therefore y_1 = \text{deflection at free end due to } 20\text{N} = \frac{W_1 a^3}{3EI} + (L-a) \times \frac{W_1 a}{2EI}$$

$$= \frac{20 \times 2^3}{3 \times 2 \times 10^{11} \times \frac{550}{6} \times 10^{-8}} + (3-2) \times \frac{20 \times 2^2}{2 \times 2 \times 10^{11} \times \frac{550}{6} \times 10^{-8}}$$

$$y_1 = 51 \times 10^{-4} \text{ m}$$

deflection at free end due to $10N$ - $\frac{WL^3}{3EI}$

$$= \frac{10 \times 0.2^3}{3 \times 2 \times 10^{11} \times \frac{\pi}{64} \times 10^{-8}}$$

$$= 4.9 \times 10^{-4} \text{ m.}$$

∴ total deflection at free end $y_B = 5.1 \times 10^{-4} + 4.9 \times 10^{-4}$

$$= 10^{-3} \text{ m. } \underline{\underline{\text{Ans}}}$$

Q.3 एक 4m लम्बी एक छेड़ी उत्तोलक धरत के स्वतंत्र सिरे पर $2WN$ एवं मध्य बिन्दु पर WN का संकेन्द्रित भार प्रयुक्त W का वह अधिकतम मान ज्ञात किजिये जिसके विचारीत होने पर धरत में 37mm के अधिकतम मान का विक्षेप न होने पाये

$$E = 2 \times 10^5 \text{ N/mm}^2 \quad I = 8 \times 10^8 \text{ mm}^4$$

Sol: ~~Ans~~ Given:-

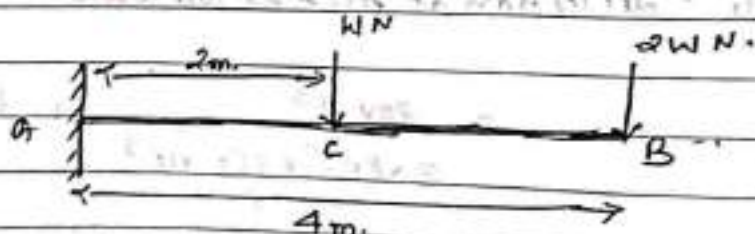
$$W_1 = 2WN$$

$$W_2 = WN$$

$$y_{max} = 37 \text{ mm} = 3.7 \times 10^{-3} \text{ m.}$$

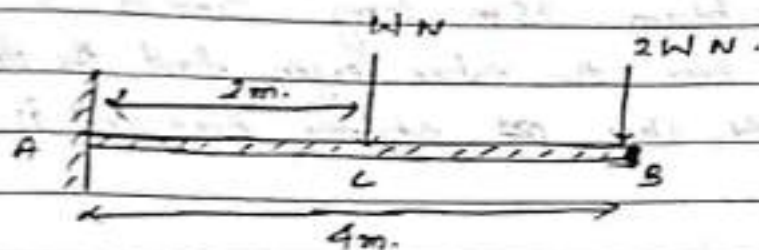
$$E = 2 \times 10^5 \text{ N/mm}^2 = 2 \times 10^{11} \text{ N/m}^2$$

$$I = 8 \times 10^8 \text{ mm}^4 = 8 \times 10^{-4} \text{ m}^4 \quad W = ?$$



max deflection will be at free end so

$$\therefore y_B = y_{\max} = 3.7 \times 10^{-3}$$



deflection at point B = $y_B = y_1 + y_2$

y_1 = deflection at free end due to WN load

$$= \frac{W \times a^3}{3EI} + (1-a) \frac{W a^2}{2EI}$$

$$= \frac{W \times 2^3}{3 \times 2 \times 10^{11} \times 8 \times 10^{-4}} + (1-0.5) \frac{W \times 2^2}{2 \times 2 \times 10^{11} \times 8 \times 10^{-4}}$$

$$y_1 = 41.67 \times 10^{-9} W$$

y_2 = deflection at free end due to $2WN$.

$$= \frac{2W \times 4^3}{3 \times 2 \times 10^{11} \times 8 \times 10^{-4}}$$

$$y_2 = 266.67 \times 10^{-9} W$$

$$\therefore \text{total deflection} = y_B = 41.67 \times 10^{-9} W + 266.67 \times 10^{-9} W$$

$$3.7 \times 10^{-3} = 308.34 \times 10^{-9} W$$

$$W = 11999.87 \text{ N} \approx 12000 \text{ N}$$

$W = 12000 \text{ N/m}$ Ans

Q:4 A cantilever beam 1.5m long carries a uniformly distributed load over the entire length find the deflection at free end if the slope at the free end is 1.5° .

Sol: Given

$l = 1.5 \text{ m}$

$\theta = 1.5^\circ = \frac{1.5 \times \pi}{180} \text{ rad}$

$y = ?$

\therefore deflection at free end $- y = \frac{W l^4}{8EI}$

$= \frac{W l^3 \times l}{8EI}$

and $\theta = \frac{1.5 \times \pi}{180} = \frac{W l^3}{6EI}$

$\frac{W l^3}{EI} = \frac{1.5 \times 6 \times \pi}{180}$

put this value in eq. (1)

$y = \frac{1.5 \times 6 \times \pi \times 1.5}{180 \times 8}$

$= 29.45 \times 10^{-3} \text{ m}$ Ans

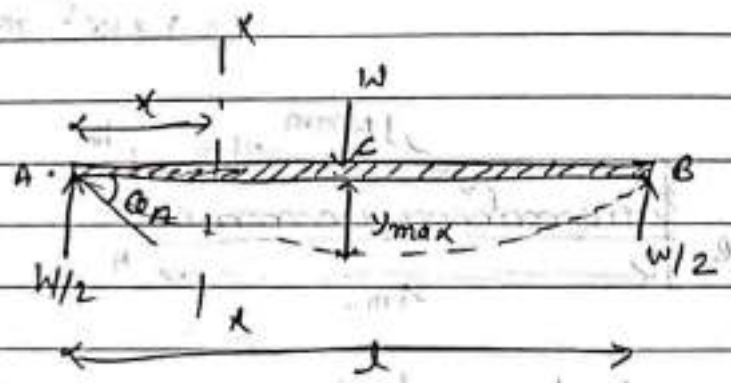
$$y_2 = 3.2 \times 10^{-3} \text{ m.}$$

∴ deflection at free end = $y = 0.133 \times 10^{-6} + 3.2 \times 10^{-3}$

$$y_1 = 3.221 \times 10^{-3} \text{ m.}$$

II Simply supported beam :-

Case I:- Simply supported beam of span l carrying a point load at mid span.



A simply supported beam AB length l carrying a point load W at mid point C .

∴ load is acting in mid point & ∴ max. deflection will occur at mid point span.

Let us take a section $X-X$ at a distance x from point A .

So Bending moment at any section XX is given by -

$$\text{B.M.} = M_x = \frac{W}{2} x$$

$$\therefore EI \frac{d^2y}{dx^2} = M_x$$

$$\therefore EI \frac{d^2y}{dx^2} = -\frac{w}{2}x$$

integrating both side we get -

$$EI \frac{dy}{dx} = -\frac{w}{4}x^2 + C_1$$

$$\text{at } x = l/2 \quad \frac{dy}{dx} = 0$$

$$\therefore 0 = -\frac{w}{4} \times \frac{l^2}{4} + C_1$$

$$C_1 = \frac{wl^2}{16}$$

$$\therefore EI \frac{dy}{dx} = -\frac{wx^2}{4} - \frac{wl^2}{16} \quad \text{--- (1)}$$

again on integrating both side

$$EI y = -\frac{wx^3}{12} - \frac{wl^2}{16}x + C_2$$

$$\text{at } x = 0 \quad y = 0$$

$$0 = 0 - 0 + C_2$$

$$\therefore C_2 = 0$$

$$\therefore EI y = -\frac{wx^3}{12} - \frac{wl^2}{16}x \quad \text{--- (2)}$$

\(\therefore\) Slope will be given by

put $x = 0$ at eq. (1)

$$EI \frac{dy}{dx} = 0 - \frac{wl^2}{16}$$

$$\frac{dy}{dx} = 0 \Rightarrow -\frac{wl^2}{16EI}$$

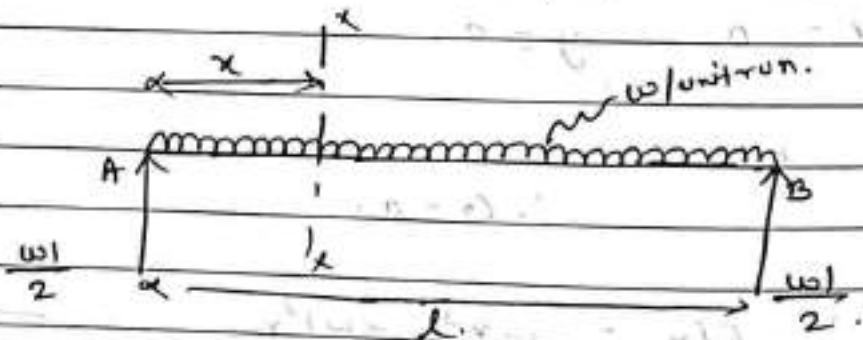
and max. deflection will be given by
put $x = l/2$ at eq. (1).

$$\begin{aligned} EI y_{\max} &= \frac{w}{12} \times \left(\frac{l}{2}\right)^3 - \frac{wl^2}{16} \\ &= \frac{wl^3}{96} - \frac{wl^3}{32} \end{aligned}$$

$$= -\frac{wl^3}{48}$$

$$\therefore y_{\max} = -\frac{wl^3}{48EI}$$

Case-II simply supported beam carrying a udl of w per unit run over the whole span -



Simply supported beam of length l carrying a udl w per unit run over the whole span.

let's take a section xx at a distance x from point A .

taking moment along section -

$$M_x = \frac{w l x x}{2} = \frac{w x^2}{2}$$

$$\therefore M_x = \frac{d^2 y}{dx^2} \times EI$$

$$\therefore EI \frac{d^2 y}{dx^2} = \frac{w l x x}{2} = \frac{w x^2}{2}$$

Integrating both side

$$EI \frac{dy}{dx} = \frac{w l x^2}{4} - \frac{w x^3}{6} + C_1 \quad \text{--- (i)}$$

the max deflection occur at mid span so slope at mid span equals zero.

$$\therefore \text{at } x = l/2 \quad \frac{dy}{dx} = 0$$

$$0 = \frac{w l x \frac{l^2}{4}}{4} - \frac{w \frac{l^3}{8}}{6} + C_1$$

$$= \frac{w l^3}{16} - \frac{w l^3}{48} + C_1$$

$$C_1 = \frac{-w l^3}{24}$$

put the value in (i),

$$\therefore EI \frac{dy}{dx} = \frac{w l x^2}{4} - \frac{w x^3}{6} - \frac{w l^3}{24} \quad \text{--- Slope eqn.}$$

\therefore slope at point A will be given by -

at $x = 0$

$$EI \left(\frac{dy}{dx} \right)_A = 0 - 0 - \frac{w l^3}{24}$$

$$\therefore \frac{d\Delta}{dx} = \frac{-w l^3}{24 E I}$$

Integrating the slope equation

$$E I y = \frac{w l x^3}{12} - \frac{w x^4}{24} - \frac{w l^3 x}{24} + C_2$$

at $x=0$ $y=0$ $\therefore C_2 = 0$

$$E I y = \frac{w l x^3}{12} - \frac{w x^4}{24} - \frac{w l^3 x}{24}$$

max deflection occur at mid span so put $x = l/2$

$$E I y = \frac{w l (l/2)^3}{12} - \frac{w (l/2)^4}{24} - \frac{w l^3 (l/2)}{24}$$

$$y_{max} = \frac{5 w l^4}{384 E I}$$

$$y_{max} = \frac{5 w l^4}{384 E I}$$

Ans

Important tables:-

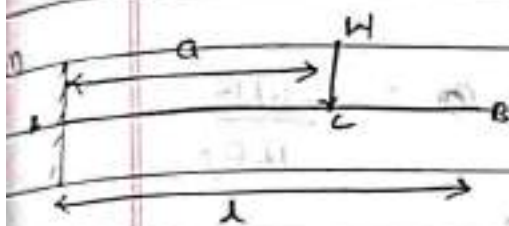
Condition

Formulae.



$$\text{slope} = -\frac{Wl^2}{2EI}$$

$$\text{deflection } y_B = -\frac{Wl^3}{3EI}$$



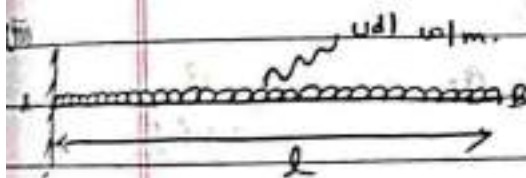
$$\text{slope} = \theta_A = \theta_B = -\frac{Wl a^2}{2EI}$$

deflection:-

$$y_C = -\frac{Wl a^3}{3EI}$$

$$y_B = y_C + (l-a)\theta_B$$

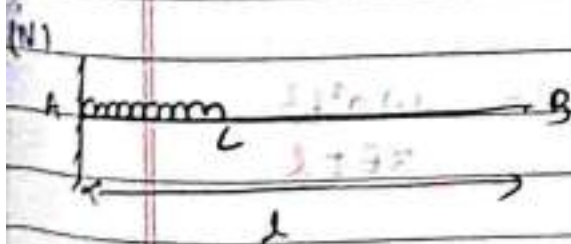
$$= -\frac{Wl a^3}{3EI} + (l-a) \times -\frac{Wl a^2}{2EI}$$



$$\text{slope} = \theta_B = -\frac{wl^3}{6EI}$$

deflection

$$y_B = -\frac{wl^4}{8EI}$$



slope:-

$$\theta_B = \theta_C = -\frac{wa^3}{6EI}$$

deflection.

$$y_C = -\frac{wa^4}{8EI}$$

$$y_B = y_C + (l-a)\theta_B$$



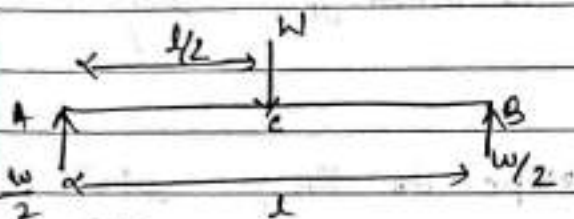
Slope :-

$$\theta_B = -\frac{ml}{EI}$$

deflection :-

$$y_B = -\frac{ml^2}{2EI}$$

(VI)



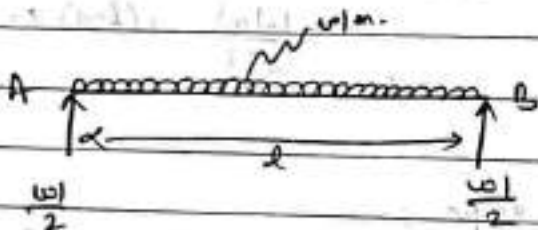
slope

$$\theta_A = \theta_B = -\frac{Wl^2}{16EI}$$

deflection

$$y_C = -\frac{Wl^3}{48EI}$$

(VII)



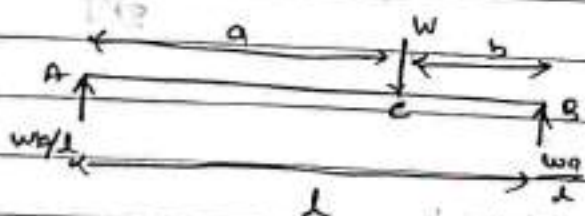
slope :-

$$\theta_A = -\frac{wl^3}{24EI}$$

deflection

$$y_{max} = -\frac{5wl^4}{384EI}$$

(VIII)



deflection :-

$$y_C = -\frac{Wla^2b^2}{3EIl}$$

$$y_{max} = -\frac{Wb(l^2 - b^2)^{3/2}}{9\sqrt{3} EIl}$$

Position of max. deflection

$$x = \sqrt{l^2 - b^2}$$

Q:1 2m लम्बी एक छेदी उत्तलक चरन के मुक्त सिरे पर 10kN का भार प्रयुक्त है।

Ans: एक छेदी उत्तलक चरन की विस्तृति 4m है। उस के प्रत्येक बिंदु पर एक समरूपित भार प्रयुक्त है। यदि चरन के सिरे पर अधिकतम ढाल 1° हो तो उस ढाल में अधिकतम विक्षेप कितना होगा।

Ans: Given:

$$l = 4m.$$

$$\theta_a = 1^\circ$$

$$\therefore 180^\circ = \pi \text{ radian}$$

$$\therefore 1^\circ = \frac{\pi}{180} \text{ radian}$$

$$\theta = \frac{wl^2}{16EI} = \frac{\pi}{180}$$

and

$$y_{max} = \frac{wl^3}{48EI} = \frac{wl^2}{16EI} \times \frac{l}{3}$$

$$= \frac{\pi}{180} \times \frac{4}{3} = 23.7 \times 10^{-3} \text{ m. } \underline{\underline{\theta}}$$

Q:2 8m लम्बी छेदी उत्तलक चरन की समस्त लम्बाई पर 10kN/m का सम वितरित भार प्रयुक्त है। यदि $E \times I = 4000 \text{ kN-m}^2$ हो तो अधिकतम ढाल विक्षेप ज्ञात करो।

Ans: Given:

$$l = 8m.$$

$$w = 10 \times 10^3 \text{ N/m.}$$

$$E \times I = 4000 \times 10^3 \text{ N-m}^2$$

$$\theta = ?$$

$$y_{max} = ?$$

$$\theta = \frac{wl^3}{24EI} = \frac{10 \times 10^3 \times 8^3}{24 \times 4000 \times 10^3}$$

$$= 0.533 \text{ radian.}$$

and deflection

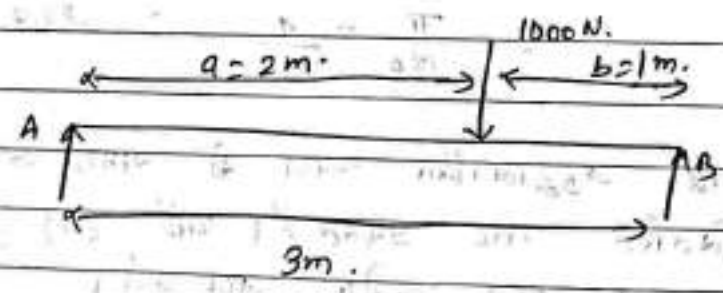
$$y_{max} = \frac{5wl^4}{384EI}$$

$$= \frac{5 \times 10 \times 10^3 \times 8^4}{384 \times 4000 \times 10^3}$$

$$y_{max} = 0.133 \text{ m. Ans}$$

Q.3 3m. विस्तार की एक क्षैप्रिक छत पर बांकी आकार में 2m. की दूरी पर एक अक्षैप्रिक भार जिसका मान 1000N है (विचारित है) अधिकतम विक्षेप का मान ज्ञात करो जब $E = 2 \times 10^5 \text{ N/mm}^2$ $I = 3 \times 10^7 \text{ m}^4$ हो।

Sol:



- Given :-
- $W = 1000 \text{ N.}$
 - $a = 2 \text{ m.} \quad b = 1 \text{ m.}$
 - $l = 3 \text{ m.}$
 - $E = 2 \times 10^5 \text{ N/mm}^2$
 - $= 2 \times 10^{11} \text{ N/m}^2$
 - $I = 3 \times 10^7 \text{ m}^4$
 - $= 3 \times 10^{15} \text{ m}^4$

max. deflection occurs at $x = \frac{l^2 - b^2}{3l}$

$$x = \frac{l^2 - b^2}{3l}$$

$$= \frac{3^2 - 1^2}{3} \text{ m} = 1.63 \text{ m from A}$$

\therefore

$$y_{\max} = \frac{wb(l^2 - b^2)^{3/2}}{9\sqrt{3}EI}$$

$$= \frac{1000 \times 1 \times (3^2 - 1^2)^{3/2}}{9\sqrt{3} \times 2 \times 10^8 \times 3 \times 10^{-5}}$$

$$= 80 \times 10^{-6} \text{ m}$$

$$y_{\max} = 80 \times 10^{-6} \text{ m} \quad \underline{\underline{Ans}}$$

Unit - 7

Buckling stresses in column & strut types componentsImportant definition:-

- (i) strut:- A bar or a member of a structure in any position and carrying an axial compressive load is called a strut.
- (ii) column:- A column is a long vertical slender bar or vertical member, say subjected to an axial compressive load.
- (iii) Slenderness ratio:- the ratio of the length of the column to the least radius of gyration is called slenderness ratio.
- (iv) Buckling factor:- It is the ratio between equivalent length of the column to the minimum radius of gyration.
- (v) Buckling load:- the minimum axial load at which the column tends to have lateral displacement or buckle is called the buckling or crippling or critical load.
- (vi) Safe load:- It is the load to which a column is actually subjected to and it is well below the buckling load.

$$\text{Safe load} = \frac{\text{buckling load}}{\text{Factor of safety}}$$

Classification of column:-

(i) Short columns:- Column which have lengths less than 8 times to their respective diameters or slenderness ratio is less than 32 are called short columns.

$$l < 8d, \text{ slenderness ratio } < 32$$

(ii) Medium columns:- Columns which have their length varying from 8 times their diameter to 30 times their diameter or their slenderness ratio lying between 32 and 120.

$$30d > l > 8d \quad \text{or} \quad \text{slenderness ratio} > 32$$

(iii) long column:- the column having their length more than 30 times to their diameter or slenderness ratio is greater than 120 known as long column.

$$l > 30d \quad \text{or} \quad \text{slenderness ratio} > 120$$

* Strength of the column depends on slenderness ratio as slenderness ratio increased compressive strength of the column decrease and tendency to buckle increased.

Equivalent length (l_e):- the equivalent length is defined as the distance between two adjacent points of contraflexure on the column.

the point of contra-

Classification of column:-

(i) Short Columns:- Column which have lengths less than 8 times to their respective diameters or slenderness ratio is less than 32. are called short columns.

$$l < 8d, \text{ slenderness ratio } < 32$$

(ii) Medium Columns:- Columns which have their length varying from 8 times their diameter to 30 times their diameter or their slenderness ratio lying between 32 and 120.

$$30d > l > 8d \quad \text{or} \quad \text{slenderness ratio} > 32$$

(iii) long column:- the column having their length more than 30 times to their diameter or slenderness ratio is greater than 120 known as long column.

$$l > 30d \quad \text{or} \quad \text{slenderness ratio} > 120$$

* Strength of the column depends on slenderness ratio as slenderness ratio increased compressive strength of the column decrease and tendency to buckle increased.

Equivalent length (l_e):- the equivalent length is defined as the distance between two adjacent points of contraflexure on the column.

the point of contra-

flexure is defined as the point on the column where there is a change in the direction of the axis of the column.

End conditions of the column:- there is 4 types of end conditions

(i) Both end hinged or pin jointed:-

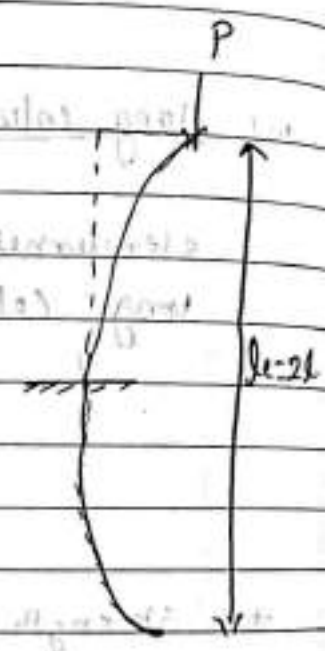
$$l_e = l$$



pin joint

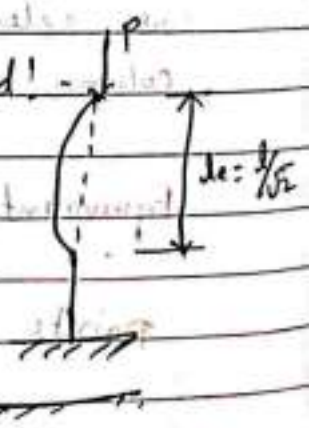
(ii) One end fixed other end free:-

$$l_e = 2l$$



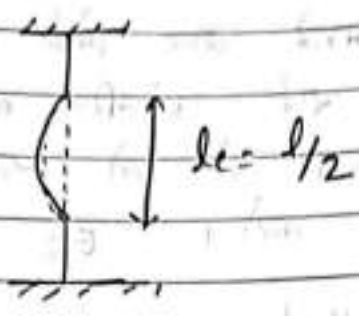
(iii) One end fixed other end pin jointed or hinged:-

$$l_e = \frac{l}{\sqrt{2}}$$



(iv) Both end fixed:

$$l_e = l/2$$



Euler's theory:-

Assumption:-

- (i) the column is initially straight and uniform lateral dimension.
- (ii) the compressive load is exactly axial.
- (iii) the material is homogeneous and isotropic.
- (iv) the weight of column is neglected.
- (v) limit of proportionality is not exceeded.

Formula:-

$$P_{euler} = \frac{\pi^2 \times E \times I_{min}}{l_e^2}$$

Where P_e = Critical load

- E = Modulus of elasticity
- l_e = equivalent length
- I_{min} = least Moment of Inertia of section

Limitation of Euler's formula:-

- (i) Euler's formula is applicable to column which are initially exactly straight and load is axial.
- (ii) this formula does not take account of the axial stress, thus the buckling load given by this formula may be much more than the actual buckling load.

Numericals:-

Q: 1

6m लंबे एक मोटे के स्तंभ का बाह्य व्यास 200mm एवं आंतरिक व्यास 180mm है। यदि स्तंभ के दोनों सिरे हिले हुए हो तो स्तंभ पर प्रयुक्त अधिकतम सुरक्षित भार ज्ञात करें। $E = 1 \times 10^5 \text{ N/mm}^2$ $Fos = 5$

Sol:

Given that

$l = 6\text{m}$

$D_o = 200\text{mm}$

$D_i = 180\text{mm}$

$E = 1 \times 10^5 \text{ N/mm}^2 = 1 \times 10^{11} \text{ N/m}^2$

$Fos = 5$

∴ End condⁿ - Both end hinged

$l_e = l = 6\text{m}$

∴ $P_{uler} = \frac{\pi^2 EI}{l_e^2}$

$I_{xx} = \frac{\pi}{64} (D_o^4 - D_i^4)$

$= \frac{\pi}{64} \times (20^4 - 18^4) = 27 \times 10^{-6} \text{ m}^4$

∴ $P_{uler} = \frac{\pi^2 \times 1 \times 10^{11} \times 27 \times 10^{-6}}{6^2}$

$= 740.22 \times 10^3 \text{ N}$

∴ Safe load = P_{uler}

$Fos = 5$
 $= 148 \text{ KN}$ Ans

एक बार की लंबाई 4m है जो एक सरल रूप से समर्थित बीम के रूप में प्रयोग की जाती है। इस बीम पर एक समान वितरित भार (UDL) का प्रयोग किया जाता है जिसकी तीव्रता 30 kN/m है। इस बीम के केंद्र पर 15mm की विक्षेपण (deflection) होती है। इस बीम को एक स्तंभ के रूप में प्रयोग करने पर निम्नलिखित शर्तों के साथ कृष्णक (crippling) भार निर्धारित करें।

Q.2 A bar of length 4m. when used as a simply supported beam and subjected to a udl of 30kN/m. over the whole span, deflected 15mm. at centre. Determine the crippling load when it is used as column with following end condⁿ

- (i) Both end pin jointed
- (ii) one end fixed other hinged
- (iii) Both end fixed.

Sol: Given :-

$$l = 4m.$$

$$w = 30kN/m.$$

$$\delta = 15mm = 0.015m.$$

∴ for simply supported beam with udl over the whole span deflection will be given by

$$\delta = \frac{5wl^4}{384EI}$$

$$5 \times 30 \times 10^3 \times 4^4$$

$$3884 \times 15 \times 10^{-3}$$

$$= 6667 \times 6.67 \times 10^6 \text{ N-m}$$

So

Euler load will be

$$(i) \text{ Both end hinged : } l_e = l$$

$$P_{\text{Euler}} = \frac{\pi^2 \times 6667 \times EI}{l_e^2}$$

$$= \frac{\pi^2 \times 6.67 \times 10^6}{4^2} \approx 4119.3 \text{ KN.}$$

(ii) One end fixed other end free - hinges

$$\therefore l_e = e l = 8 \frac{l}{\sqrt{2}} = 5.656 l$$

$$\therefore P_{uler} = \frac{\pi^2 \times 6.67 \times 10^6}{2.89^2}$$

$$= 8219.6 \text{ kN}$$

(iii) Both end fixed: $l_e = \frac{l}{2} = 2 \text{ m}$

$$\therefore P_{uler} = \frac{\pi^2 \times 6.67 \times 10^6}{2^2}$$

$$= 16.47 \times 10^6 \text{ N} \quad \underline{\underline{Ans}}$$

Q: Determine the ratio of the buckling strength of ~~the~~ ^{two} columns of circular cross-section one hollow and other solid when both are made of the same material, have the same length, cross-sectional area and end conditions. The internal diameter of the hollow column is half of its external diameter.

Sol:

Given:-

Area of solid column = Area of hollow column
 $A_s = A_h$

$$\frac{\pi}{4} \times D_s^2 = \frac{\pi}{4} (D_h^2 - d_h^2)$$

$$\frac{\pi}{4} \times D_s^2 = \frac{\pi}{4} (D_h^2 - (\frac{D_h}{2})^2) \quad d_h = \frac{D_h}{2} \text{ (Given)}$$

$$D_s^2 = D_h^2 - \frac{D_h^2}{4}$$

$$D_s^2 = \frac{4D_H^4 - D_H^4}{4}$$

$$D_s^2 = \frac{3D_H^4}{4}$$

$$D_s = 1.866D_H$$

∴ **B**

buckling strength of hollow column - $P_H = \frac{\pi^2 E I_H}{l^2}$

buckling strength of solid column $P_S = \frac{\pi^2 E I_S}{l^2}$

$$\frac{P_H}{P_S} = \frac{\pi^2 E I_H}{l^2} \times \frac{l^4}{\pi^2 E I_S}$$

$$\left(\frac{P_H}{P_S}\right) \times \frac{I_S}{I_H} = \frac{I_H}{I_S} = \frac{\frac{\pi}{64} \times (D_H^4 - d_H^4)}{\frac{\pi}{64} \times D_S^4}$$

$$\frac{P_H}{P_S} \times \frac{I_S}{I_H} = \frac{D_H^4 - \frac{D_H^4}{16}}{D_S^4} = \frac{15}{16} \frac{D_H^4}{D_S^4}$$

$$\frac{P_H}{P_S} \times \frac{I_S}{I_H} = \frac{.9375 D_H^4}{D_S^4}$$

$$= \frac{.9375 \times D_H^4}{(1.866)^4 \times D_H^4}$$

$$= 1.66$$

$$\therefore \frac{P_H}{P_S} = 1.66 \quad \underline{A_u}$$

1st SEM - "ME 01" & 5th SEM - "ME 02"
SEM 10:30-11:30

Rankine's Hypothesis for columns

As per Rankine's hypothesis -

$$\frac{1}{P_{Rankine}} = \frac{1}{P_c} + \frac{1}{P_e}$$

P_e = Euler load
 P_c = crushing load

Ans:
$$\frac{1}{P_R} = \frac{1}{\sigma_c A} + \frac{1}{\pi^2 EA \left(\frac{k}{l_e}\right)^2}$$

on solving

$$P_{Rankine} = \frac{\sigma_c A}{1 + a \left(\frac{l_e}{k}\right)^2}$$

where:

- σ_c = crushing stress
- A = Area
- a = const.
- l_e = equivalent length
- k = Radius of gyration

Numerical:-

Q: एक लंबा कोष्ठ का बरोबरता क्षेत्रफल, जिसके दोनों सिरे मोड़ हैं, 5 m लंबा है। इसका बाहरी व्यास 120 mm और धातु की मोटाई 15 mm है। यदि लंबीय प्रतिबल = 50 N/mm² और रेखांक स्थिरांक $a = \frac{1}{1600}$ हो तो खंभे पर प्रयुक्त सुरक्षित भार का मान निर्धारित करें। सुरक्षा गुणक = 5.

Given:

$$l = 5\text{m.}$$

$$D = 120\text{mm.}$$

$$= .12\text{m.}$$

$$t = 15\text{mm.}$$

$$d = D - 2t$$

$$= 1015\text{mm.}$$

$$= 101.09\text{m.}$$

$$G_c = 560\text{N/mm}^2$$

$$= 560 \times 10^6\text{N/m}^2$$

$$a = \frac{1}{1600}$$

$$F.O.S. = 5$$

$$\therefore A = \frac{\pi}{4} (D^2 - d^2) = 4.94 \times 10^{-3}\text{m}^2$$

$$P_R = \frac{G_c \times A}{1 + a \left(\frac{le}{k}\right)^2}$$

①

$$M.O.I. = I = \frac{\pi}{64} (D^4 - d^4)$$

$$= 6.95 \times 10^{-6}\text{m}^4$$

$$= \frac{560 \times 10^6 \times 4.94 \times 10^{-3}}{1 + \frac{1}{1600} \left(\frac{2.5}{.0375}\right)^2}$$

$$I = A k^2$$

$$k = A \cdot 0.0375$$

 \therefore both end fixed - ②

$$\text{so } le = \frac{75 \cdot l}{2} = 2.5\text{m.}$$

$$P_R = \frac{G_c \times A}{1 + a \left(\frac{le}{k}\right)^2} = \frac{560 \times 10^6 \times 4.94 \times 10^{-3}}{1 + \frac{1}{1600} \left(\frac{2.5}{.0375}\right)^2}$$

$$P_R = 732282.35$$

$$\therefore \text{ Safe load } = \frac{P_R}{F.O.S.} = 146456.4\text{N.}$$

$$= 146.4\text{KN.}$$

Q.1. Given:-

$$l = 5\text{m.}$$

$$D = 120\text{mm.}$$

$$= .12\text{m.}$$

$$t = 15\text{mm.}$$

$$= .015\text{m.}$$

$$d = D - 2t$$

$$G_c = 560\text{N/mm}^2$$

$$= 560 \times 10^6\text{N/m}^2$$

$$a = \frac{1}{1600}$$

$$F.O.S. = 5$$

$$\therefore A = \frac{\pi}{4} (D^2 - d^2) = 4.94 \times 10^{-3}\text{m}^2$$

$$P_R = \frac{G_c \times A}{1 + a \left(\frac{le}{K}\right)^2}$$

$$M.O.I. = I = \frac{\pi}{64} (D^4 - d^4)$$

$$= 6.95 \times 10^{-6}\text{m}^4$$

$$= \frac{560 \times 10^6 \times 4.94 \times 10^{-3}}{1 + \frac{1}{1600} \left(\frac{2.5}{.0375}\right)^2}$$

$$I = AK^2$$

$$K = .0375$$

\(\because\) both end fixed -

$$\text{so } \frac{le}{K} = \frac{2.5 \times 2}{2} = 2.5\text{m.}$$

$$P_R = \frac{G_c \times A}{1 + a \left(\frac{le}{K}\right)^2} = \frac{560 \times 10^6 \times 4.94 \times 10^{-3}}{1 + \frac{1}{1600} \left(\frac{2.5}{.0375}\right)^2}$$

$$P_R = 732282.35$$

$$\therefore \text{Safe load} = \frac{P_R}{F.O.S.} = 146456.4\text{N.}$$

$$= 146.4\text{KN. } \underline{\underline{A}}$$

Q:- 3m. लम्बे स्तूप की एक खोखली नली के सिरे केंद्र से एक रेडियन स्तर से एक एक प्रांत आंकुचन कार की तुलना करो। इस नली का बाह्य व्यास 50mm. और चपटा की मोटाई 10mm. है दोनों सिरे हिन्ज है।

$$P = 2 \times 10^5 \text{ N/mm}^2 \quad G_c = 320 \text{ N/mm}^2 \quad \text{और } a = \frac{1}{7500}$$

Sol:- Given

$$l = 3 \text{ m.}$$

$$P = 2 \times 10^{11} \text{ N/m}^2$$

$$G_c = 320 \times 10^6 \text{ N/m}^2$$

∴ both end hinged ∴ $l_e = l$

$$= 3 \text{ m.}$$

$$a = \frac{1}{7500}$$

$$D = 50 \text{ mm} = 0.05 \text{ m.}$$

$$t = 10 \text{ mm} = 0.01 \text{ m.}$$

$$\therefore d = D - 2t = 0.03 \text{ m.}$$

$$\text{Area} = A = \frac{\pi}{4} \times (D^2 - d^2)$$

$$= 1.256 \times 10^{-3} \text{ m}^2$$

$$\text{Moment of Inertia} = I = \frac{\pi}{64} \times (D^4 - d^4)$$

$$= \frac{\pi}{64} \times (0.05^4 - 0.03^4)$$

$$I = 9.67 \times 10^{-7} \text{ m}^4$$

$$\therefore I = k^2 \times A$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{9.67 \times 10^{-7}}{1.256 \times 10^{-3}}} = 0.0146 \text{ m}$$

$$\therefore \text{Euler load } P_{\text{Euler}} = \frac{\pi^2 \times E \times I}{L_e^2}$$

$$= \frac{\pi^2 \times 2 \times 10^{11} \times 0.67 \times 10^{-7}}{3^2}$$

$$P_{\text{Euler}} = 58559.6 \text{ N}$$

$$\text{and Rankine load} = \frac{G_c \times A}{1 + a \left(\frac{L_e}{k}\right)^2}$$



$$= \frac{320 \times 10^6 \times 1.256 \times 10^{-3}}{1 + \frac{1}{7500} \times \left(\frac{3}{0.0146}\right)^2}$$

$$P_{\text{Rankine}} = 60625.31 \text{ N}$$

$$P_{\text{Euler}} = 58559.6$$

$$P_{\text{Rankine}} = 60625.31$$

Ans

